Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics A 967 (2017) 433–436

NUCLEAR PHYSICS A

www.elsevier.com/locate/nuclphysa

Optimized fluid dynamics for heavy ion collisions☆

Dennis Bazow[a], Ulrich Heinz[a], Michael Strickland[b]

[a]*Department of Physics, The Ohio State University, Columbus, Ohio 43210-1117, USA*
[b]*Department of Physics, Kent State University, Kent, Ohio 44242, USA*

Abstract

The (viscous) anisotropic hydrodynamic approach, especially after perturbative inclusion of all residual viscous terms, has been shown to dramatically outperform viscous hydrodynamics in several simplified situations for which exact solutions exist but which share with realistic expansion scenarios the problem of large dissipative currents. We will report on the present status of applying viscous anisotropic hydrodynamics in a highly efficient simulation of the full three-dimensional quark-gluon plasma. Results from accelerated 3+1-dimensional viscous hydrodynamic simulations using graphics processing units will be compared to the anisotropic frameworks.

Keywords:
Relativistic fluid dynamics, Quark-gluon plasma, Anisotropic hydrodynamics, GPU

1. Introduction

Quantitative modeling of relativistic heavy ion collisions has dramatically improved in recent years. Simulations include an initial state and pre-equilibrium model for the first $\sim 0.5 - 1$ fm/c when the quark-gluon plasma (QGP) is highly momentum anisotropic and far from thermalized, followed by viscous hydrodynamic simulations of the medium for the semi-isotropic and semi-thermal QGP for the next $O(10)$ fm/c, and finally a microscopic hadronic treatment of the reformed hadrons. One of the main uncertainties in this chain is from the early evolution stage when the QGP is highly anisotropic and cannot be treated with viscous hydrodynamics. This uncertainty is usually encapsulated in model parameters describing the initial state and early pre-equilibrium dynamics. To constrain these and other model parameters with the help of experimental observables, these simulations are coupled with advanced statistical methods based on Bayesian statistics [1, 2]. Since it is very computationally expensive to train these models to cover very high-dimensional parameter spaces we optimize the simulations along two fronts: we (1) speed up the 3+1 dimensional fluid dynamic simulation by performing the calculations on graphics processing units (GPUs) [3] and (2) limit the uncertainty from the early evolution stage by using the viscous anisotropic hydrodynamic formalism [4] which accounts for the large momentum anisotropies at early times non-perturbatively and thus allows one to start the hydrodynamic stage earlier. The viscous anisotropic formalism improves

☆Supported by the US Department of Energy under awards DE-SC0004286 and DE-SC0013470 and by the National Science Foundation through the JETSCAPE Collaboration (award number ACI-1550223).

http://dx.doi.org/10.1016/j.nuclphysa.2017.06.028

Number of grid points	C/CPU (ms/step)	CUDA/GPU (ms/step)	Speedup
$128 \times 128 \times 32$	7145.978	63.261	112.960
$128 \times 128 \times 64$	13937.896	123.527	112.833
$128 \times 128 \times 128$	30717.367	244.450	125.659
$256 \times 256 \times 32$	25934.547	236.593	109.617
$256 \times 256 \times 64$	57387.141	472.391	121.482
$256 \times 256 \times 128$	129239.959	939.340	137.586

Table 1. Performance results of the C/CPU and CUDA/GPU versions of CPU-VH and GPU-VH by measuring the computer time it takes to complete one full RK step, averaged over 100 time steps, at different spatial resolutions.

upon leading-order anisotropic hydrodynamics (see [5] for a review) by also including previously neglected residual components of the shear stress tensor through Isreal-Stewart-like perturbative transport equations. The combination of these advances leads to faster and quantitatively more reliable dynamical simulations of heavy ion collisions with fewer parameters.

2. GPU-accelerated (3+1)-dimensional second-order viscous hydrodynamics

Our implementation of second-order viscous relativistic fluid dynamics on graphics processing units (GPU-VH) is described in [3]. This code has since been adapted to evolve the second-order anisotropic hydrodynamic equations of motion, but not yet on GPUs. The performance of GPU-VH, at different spatial resolutions, is measured via the time it takes to complete 100 full time steps. Table 1 compares performance of GPU-VH on the GeForce GTX 980 Ti graphics card relative to the CPU-VH code run on the host machine with a 2.6 GHz Intel Xeon CPU E5-2697 v3 using a single core. We observe speed-up factors of $\mathcal{O}(100)$.

3. (3+1)-dimensional second-order anisotropic hydrodynamics

Assuming negligible net baryon density, relativistic fluid dynamics is described by the conservation laws for energy and momentum, $\partial_\mu T^{\mu\nu}(x) = 0$, complemented by relaxation-type evolution equations for the dissipative flows. For anisotropic systems $T^{\mu\nu}$ can be decomposed with respect to the fluid four-velocity u^μ and the space-like four-vector z^μ (defining the direction of the largest anisotropy, which for heavy-ion collisions is the beam direction [5]), parametrized by $u^\mu \equiv (u^0, u^1, u^2, u^3) = (u_\tau, \vec{u}_\perp, u_\eta)$ and $z^\mu = \gamma_z(\tau u^3, 0, 0, u_\tau/\tau)$, where $\gamma_z^{-2} \equiv 1 + u_\perp^2$. Identifying the energy density $\mathcal{E}$ with its equilibrium form via Landau matching and demanding that the longitudinal pressure in the direction of the anisotropy is equal to its "anisotropic equilibrium" value [6], we can decompose the energy-momentum tensor as (indicating "anisotropic equilibrium" quantities with an over-hat and $\tilde{\mathcal{O}} = \mathcal{O} - \hat{\mathcal{O}}$) [7]:

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu - \mathcal{P}_\perp \Delta_\perp^{\mu\nu} + \hat{\mathcal{P}}_L z^\mu z^\nu + 2\tilde{W}_{\perp z}^{(\mu} z^{\nu)} + \tilde{\pi}_\perp^{\mu\nu} \,. \tag{1}$$

Here, $\hat{\mathcal{P}}_L$ is the total longitudinal pressure, the transverse pressure $\mathcal{P}_\perp \equiv \hat{\mathcal{P}}_\perp + 3\tilde{\Pi}/2$ is the sum of the "anisotropic equilibrium" pressure $\hat{\mathcal{P}}_\perp$ and the residual bulk viscous pressure $\tilde{\Pi}$, $\tilde{W}_{\perp z}^\mu \equiv -\Delta_{\perp,\alpha}^\mu T^{\alpha\beta} z_\beta$ is the energy-momentum diffusion current in the z direction, and the transverse shear stress tensor is $\tilde{\pi}_\perp^{\mu\nu} \equiv T^{\langle\mu\nu\rangle}$. The transverse projection tensor $\Delta_\perp^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu + z^\mu z^\nu$ is used to project four-vectors and tensors into the space orthogonal to u^μ and z^μ. By construction, the dissipative terms satisfy the constraints $u_\mu \tilde{W}_{\perp z}^\mu = z_\mu \tilde{W}_{\perp z}^\mu = u_\mu \tilde{\pi}_\perp^{\mu\nu} = z_\mu \tilde{\pi}_\perp^{\mu\nu} = g_{\mu\nu}\pi^{\mu\nu} \equiv 0$.

To close the conservation laws, additional evolution equations for $\hat{\mathcal{P}}_L$ and $\tilde{\Pi}$ (entering in Eq. (1) through $\mathcal{P}_\perp$), and for the residual dissipative currents $\tilde{W}_{\perp z}^\mu$ and $\tilde{\pi}_\perp^{\mu\nu}$ must be provided [7, 8]. In addition, we must determine $\hat{\mathcal{P}}_\perp$. In kinetic theory

$$\hat{\mathcal{P}}_\perp \equiv -\frac{1}{2}\Delta_\perp^{\mu\nu}\hat{T}_{\mu\nu} = \frac{1}{2}\left(\mathcal{E} - \hat{\mathcal{P}}_L - \bar{\mathcal{R}}(\xi)(\mathcal{E} - 3\mathcal{P}_0)\right) \,, \tag{2}$$

where the last term is the trace of the energy-momentum tensor. The $\bar{\mathcal{R}}$-function above depends only on the leading order anisotropy of the system (measured by the macroscopic quantities $\mathcal{E}/\hat{\mathcal{P}}_L$); we take the