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# On the cancellation of Newtonian singularities in higher-derivative gravity

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ABSTRACT

Recently there has been a growing interest in quantum gravity theories with more than four derivatives, including both their quantum and classical aspects. In this work we extend the recent results concerning the non-singularity of the modified Newtonian potential to the most relevant case in which the propagator has complex poles. The model we consider is Einstein–Hilbert action augmented by curvature-squared higher-derivative terms which contain polynomials on the d'Alembert operator. We show that the classical potential of these theories is a real quantity and it is regular at the origin despite the (complex or real) nature or the multiplicity of the massive poles. The expression for the potential is explicitly derived for some interesting particular cases. Finally, the issue of the mechanism behind the cancellation of the singularity is discussed; specifically we argue that the regularity of the potential can hold even if the number of massive tensor modes and scalar ones is not the same.



## 1. Introduction

The twentieth century has brought two fundamental ideas to Physics: the curved space–time and the quantization of matter. In spite of the great success each of these insights has achieved owed to the outstanding experimental verification of general relativity and quantum field theory, no fully consistent way of combining both paradigmatic theories is known. Since the 1960's it is known that the renormalization of quantum fields on curved space–time using perturbative methods requires introducing the curvature-squared terms $R^2, R^2_{\mu\nu}$ and $R^2_{\mu\nu\alpha\beta}$ which violate the unitarity of the theory [1]. The situation is no better when gravity itself is quantized — for example, the gravity model with fourth-derivative terms is renormalizable, but has negative-norm states [2]. Conciliating unitarity and renormalizability is one of the main problems in quantum gravity and has motivated the search for theories which relied on fundamentally different basic principles, such as string theory.

Nonetheless, within the framework of standard quantum field theory, the introduction of terms of order two in curvature but with more than four derivatives has shown to make the theory superrenormalizable [3] and also allowed for the possibility of providing a unitary S-matrix [4]. In fact, in this case the associated propagator may admit massive complex poles; such virtual modes would have complex kinetic energy, being unstable and leading to a unitary theory *à la* Lee–Wick [4,5].

Renormalizability in gravity might be related to the behaviour of the classical interparticle potential of the model [6]; indeed there is a conjecture which states that renormalizable gravity theories have a finite non-relativistic potential at the origin [7]. This relation was first noticed in Stelle's seminal works [2,8] which showed that the fourth-derivative gravity is renormalizable, and has a regular potential. More recently, there have been interesting investigations on this conjecture in massive gravity models, and also in theories with dimensions different than four (see [7] and references therein).

In a recent paper [6] Modesto, Paula Netto and Shapiro moved a step further on this discussion with the generalization of Stelle's result so as to account for a class of superrenormalizable particular cases of the model defined by the action[1] [3]

[1] The cosmological constant is omitted because it does not affect the regularity of the classical potential, besides being very small. Of course, the corresponding term is necessary to the renormalization of the theory. Our sign convention is to define the Minkowski metric as $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, while the Riemann and Ricci tensors are $R^\rho{}_{\lambda\mu\nu} = \partial_\mu \Gamma^\rho{}_{\lambda\nu} + \cdots$ and $R_{\mu\nu} = R^\rho{}_{\mu\nu\rho}$. To simplify notation we set $\hbar = c = 1$.

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$$\mathcal{S}_{\text{grav}} = \int d^4x\sqrt{-g}\left(\frac{2}{\kappa^2}R + R\tilde{F}_1^{(a)}(\Box)R + R_{\mu\nu}\tilde{F}_2^{(b)}(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\tilde{F}_3^{(c)}(\Box)R^{\mu\nu\alpha\beta}\right), \tag{1}$$

where $\kappa^2 = 32\pi G$ and $\tilde{F}_i^{(j)}(\Box)$ are polynomials of degree $j \geq 0$ on the d'Alembert operator. We recall that the condition $a = b = c$ makes the model renormalizable,[2] and that it becomes superrenormalizable if $a = b = c \geq 1$. The analysis in [6] was restricted to the (super)renormalizable case with the additional constraint that the polynomials $\tilde{F}_i^{(j)}$ yielded only simple, real poles in the propagator. The main result they obtained was another verification of the aforementioned conjecture, showing that this model has a finite classical potential at the origin: the massive tensor and scalar modes contribute in such a manner to precisely cancel out the Newtonian singularity. Also, it was suggested that this happens not only because of a particular balance between the attractive forces owed by the healthy modes and the repulsive ones related to the ghosts, but also due to a specific matching of the number of tensor and scalar modes. Namely, they conjectured that if the number of these massive excitations is not the same (which is related to having polynomials $\tilde{F}_i^{(j)}$ of different orders and, therefore, losing renormalizability), the potential would not be regular.

In the present work we extend the result of [6] to the most general and more interesting case in which the massive poles of the propagator are complex, and may have degeneracies. We show that the action (1) always yields a real, regular potential at the origin — disregard the number, the nature (complex or real) and order of the massive poles. As a consequence, the mechanism which allows the cancellation of the Newtonian singularity is broader than the one proposed in [6].

To close this introductory section, it is worthwhile to mention that there exists a connection between the polynomial action (1) and the infinite-derivative "ghost free" gravity [9] (see, for instance, Refs. [10] for recent studies on singularities in the latter model). In fact, it was argued in [11] that the quantum corrections to the non-local classical model would lead to an infinite amount of complex ghost-like states, making the study of the polynomial gravity mandatory — with a special interest to the case of complex poles. In particular, it was conjectured that the cancellation of the Newtonian singularity in the non-local model could be owed to the effect of an infinite number of "hidden" complex excitations [11]; the present work can be viewed as a step towards this result. Finally, it is good to remember that singularities (of both black hole type and cosmological) constitute a central topic in gravitational physics and one of the main reasons for quantum gravity. Hence it is important to explore the classical singularities in the new promising quantum gravity model (1). The influence of fourth derivatives has already been investigated in the context of black holes [12] and cosmological solutions [13]. Inasmuch as the general polynomial theory is more complicated than Stelle's gravity, it is sound to start from the Newtonian case, as it is done in the present work. Other investigations on the low-energy phenomenology of (super)renormalizable higher-derivative local theories are carried out in [14] and in the parallel works [15,16].

## 2. Real potential with complex poles

The classical potential of a gravitational theory is computed by considering the metric to be a small fluctuation around the flat space–time, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa^2 h_{\mu\nu}$, and approximating the geometric quantities by their linearized forms. The quadratic terms in the Riemann tensor need not to be considered in the linear approximation, because the relation ($p \in \mathbb{N}$)

$$\int d^4x\sqrt{-g}\left(R\Box^p R - 4R_{\mu\nu}\Box^p R^{\mu\nu} + R_{\mu\nu\alpha\beta}\Box^p R^{\mu\nu\alpha\beta}\right) = \mathcal{O}(h^3)$$

means that at this level there are only two independent quantities among the scalars $R\Box^p R$, $R_{\mu\nu}\Box^p R^{\mu\nu}$ and $R_{\mu\nu\alpha\beta}\Box^p R^{\mu\nu\alpha\beta}$ (see, e.g., [3]). Hence, we may substitute the polynomials $\tilde{F}_i^{(j)}$ by $F_1^{(p)} \equiv \tilde{F}_1^{(a)} - \tilde{F}_3^{(c)}$, $F_2^{(q)} \equiv \tilde{F}_2^{(b)} + 4\tilde{F}_3^{(c)}$ and $F_3 \equiv 0$, which simplifies the Lagrangian associated with the action (1) leading to

$$\mathcal{L}_{\text{grav}} = \sqrt{-g}\left(\frac{2}{\kappa^2}R + RF_1^{(p)}(\Box)R + R_{\mu\nu}F_2^{(q)}(\Box)R^{\mu\nu}\right), \tag{2}$$

where $p = \max\{a, c\}$ and $q = \max\{b, c\}$.

We note that, via the substitution $\partial_\mu \longmapsto -ik_\mu$, each $F_i^{(j)}(\Box)$ corresponds to a polynomial $F_i^{(j)}(-k^2)$ in the momentum space representation. Let us now define the polynomials

$$\begin{aligned} Q_0(k^2) &= 1 - \kappa^2 k^2\left[F_2^{(q)}(-k^2) + 3F_1^{(p)}(-k^2)\right], \\ Q_2(k^2) &= 1 + \frac{\kappa^2 k^2}{2}F_2^{(q)}(-k^2), \end{aligned} \tag{3}$$

respectively of order $n_0 = 1 + \max\{p, q\}$ and $n_2 = q + 1$ on $k^2$. It is not difficult to verify that in the de Donder gauge the momentum space representation of the propagator associated to (2) is given in terms of $Q_0$ and $Q_2$ as

$$D = \frac{1}{k^2 Q_2}P^{(2)} - \frac{1}{2k^2 Q_0}P^{(0-s)} + \frac{2\lambda}{k^2}P^{(1)} + \left[-\frac{3}{2k^2 Q_0} + \frac{4\lambda}{k^2}\right]P^{(0-w)} + \frac{\sqrt{3}}{2k^2 Q_0}\left[P^{(0-sw)} + P^{(0-ws)}\right]. \tag{4}$$

Here $\lambda$ is a gauge parameter, and $P^{(2)}$, $P^{(0-s)}$, etc. are the usual Barnes–Rivers operators [17], whose indices have been omitted. The masses of the propagated fields correspond to the poles of (4), which turn out to be the roots of $Q_{0,2}$. According to the fundamental theorem of algebra, there are $n_0$ massive modes of spin-0, and $n_2$ massive spin-2 modes (complex roots and degeneracies may occur depending on the coefficients of the polynomials). Therefore, it is more useful to rewrite these polynomials in the factored form

$$Q_i(k^2) = \frac{(m_{(i)1}^2 - k^2)(m_{(i)2}^2 - k^2)\cdots(m_{(i)n_i}^2 - k^2)}{m_{(i)1}^2 m_{(i)2}^2 \cdots m_{(i)n_i}^2}. \tag{5}$$

The poles of the propagator (4) are defined as $m_{(i)j}^2$, or $\pm m_{(i)j}$ if we consider that the polynomial is on $k$. The index $i = 0, 2$ between parentheses labels the spin of the particle associated to the $j$th excitation.

The field $h_{\mu\nu}$ generated by a point-like mass $M$ in rest, $T_{\mu\nu}(\mathbf{r}) = M\eta_{\mu 0}\eta_{\nu 0}\delta^{(3)}(\mathbf{r})$, can be evaluated by means of the Fourier transform method [5,6] or via an auxiliary field formulation as in [15]. The classical potential $\phi$ is then proportional to $h_{00}$, namely, $\phi = \frac{\kappa^2}{2}h_{00}$. It can also be computed following the scheme of [18]. All in all, it is possible to show that the (modified) Newtonian potential is given by

$$\phi(r) = -\frac{iGM}{\pi r}\int_{-\infty}^{+\infty}\frac{dk}{k}e^{ikr}\left(\frac{4}{3Q_2(-k^2)} - \frac{1}{3Q_0(-k^2)}\right). \tag{6}$$

[2] The case $a = b = c = 0$ corresponds to Stelle's renormalizable model [2].