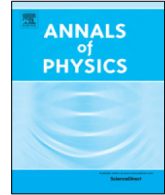




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# Classification of matrix product states with a local (gauge) symmetry



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### HIGHLIGHTS

- Structure of tensors generating states (MPS) with local symmetry is determined.
- Three types of states analyzed: matter, pure gauge field and their combined state.
- Structure of gauge field Hilbert space derived from local symmetry assumption.
- New gauge invariant states described, not the result of gauging a global symmetry.

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### ABSTRACT

Matrix Product States (MPS) are a particular type of one dimensional tensor network states, that have been applied to the study of numerous quantum many body problems. One of their key features is the possibility to describe and encode symmetries on the level of a single building block (tensor), and hence they provide a natural playground for the study of symmetric systems. In particular, recent works have proposed to use MPS (and higher dimensional tensor networks) for the study of systems with local symmetry that appear in the context of gauge theories. In this work we classify MPS which exhibit local invariance under arbitrary gauge groups. We study the respective tensors and their structure, revealing known constructions that follow known gauging procedures, as well as different, other types of possible gauge invariant states.

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## 1. Introduction

Gauge theories play a paramount role in modern physics. Through the gauge principle, the theories describing the fundamental interactions in the standard model of particle physics are obtained by lifting the global symmetries of the interaction-free matter theories to be local symmetries, minimally coupled [1] to a gauge field. Moreover, they also emerge as effective low-energy descriptions in several condensed matter scenarios [2]. Historically, the gauging procedure was first conceived as a transformation of a Lagrangian or Hamiltonian describing a physical system; however, it can be performed on the level of quantum states as well, irrespective of dynamics associated to a specific theory.

In spite of their central role in the standard model, non-Abelian gauge theories still involve puzzles to be solved. Their complete understanding still poses a significant challenge due to non-perturbative phenomena (e.g. low energy QCD). Among the various approaches proposed to tackle the strongly coupled regime, a particularly general and successful one is lattice gauge theory [3]. Monte Carlo sampling of Wilson's Euclidean lattice version of gauge theories has so far been the most successful method of numerical simulation, nevertheless, it suffers from its own drawbacks. The sign problem [4] prevents application to systems with large fermionic densities, and the use of Euclidean time does not allow to study real time evolution and non-equilibrium phenomena in general scenarios. In order to describe real-time evolution of such theories, one is forced to abandon the Monte Carlo approach, and search for other methods. In this context, the Hamiltonian formulation of Kogut and Susskind [5] has been receiving renewed interest, with two recent approaches coming from the quantum information and quantum optics community: quantum simulation, using optical, atomic or solid-state systems [6,7], and tensor network states.

The representation of quantum many-body states as tensor networks is connected to White's density-matrix renormalization group [8], and in the case of one dimensional spin lattices is known as matrix product states (MPS) [9]. Among many useful properties of tensor networks, one which makes them well suited to the description of states with symmetries, is the ability to encode the symmetry on the level of a single tensor (or a few) describing the state. In the case of global symmetries, both for MPS and for certain classes of PEPS in 2D (Projected Entangled Pair States – the generalization of MPS to higher dimensional lattices), the relation between the symmetry of the state and the properties of the tensor is well understood [10]. Tensor networks studies of lattice gauge theories have so far included numerical works (e.g., mass spectra, thermal states, real time dynamics and string breaking, phase diagrams etc. for the Schwinger model and others) [11–30], furthermore, several theoretical formulations of classes of gauge invariant tensor network states have been proposed [31–35]. In all of the latter the construction method follows the ones common to conventional gauge theory formulations: symmetric tensors are used to describe the matter degree of freedom, and later on a gauge field degree of freedom is added, or, alternatively – a pure gauge field theory is considered. While the usefulness of tensor networks in lattice gauge theories has certainly been demonstrated by the above mentioned works, so far there were few attempts (e.g. [13]) to generally classify tensor network states with local symmetry.

In this paper, starting from the assumption of a local symmetry, we find necessary and sufficient conditions to be satisfied by the tensors encoding a MPS. Similar work was done in [13] for MPS with local  $U(1)$  symmetry and with open boundary conditions. We focus on translation-invariant MPS, and deal with arbitrary finite or compact Lie groups. Clearly, one could come up with arbitrarily complicated constructions of states with a local symmetry (e.g. by using many kinds of symmetric tensors). Our analysis is therefore limited to three physically meaningful settings corresponding to: states describing matter, pure gauge field states and states of both matter and gauge field. In our analysis the matter degrees of freedom are represented by “spins”; this could in principle be extended to fermionic systems, and in particular to Majorana fermions.

For states describing only matter we find that local symmetries can only be trivial, and show how to gauge such states by adding another degree of freedom. When investigating pure gauge states we show that local symmetry in MPS requires a specific structure of the Hilbert space describing the gauge field degree of freedom. In Wilson's lattice gauge theories, in order to obtain minimal coupling in a continuum limit, the gauge field degree of freedom is set as a group element in the same

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