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Dynamics and scaling properties for a one-dimensional impact system with two periodically vibrating walls

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ABSTRACT

We investigate the dynamics of a system composed of a particle suffering impacts between two heavy periodically vibrating walls. An original, nonlinear area preserving mapping is obtained. The control parameters of amplitude of perturbation and frequency of oscillation play an important role in the phase space, shaping the portion of chaotic seas, position of invariant curves and the amount of KAM islands. The study of the behavior of the root mean square velocity was made via analytical description and numerical simulations. We proposed scaling arguments to describe its dynamics and our results show remarkably good agreement between the theory and the simulations concerning a scaling invariance with respect to the control parameters. Also, an analysis of the diffusion coefficient confirms the validity of the scaling invariance, giving robustness to our modeling.

1. Introduction

Hamiltonian dynamical systems with moving boundaries have been studied for a long time [1–3], where their dynamics are typically non-ergodic and non-integrable. These systems present mixed dynamics in the phase space, with KAM islands, invariant tori, spanning curves and chaotic seas, where the interface between them is very complex and not yet fully understood. Also, depending on both of the initial conditions as well as control parameters, such systems may present very rich and hence complex dynamics, therefore leading to a huge variety of nonlinear phenomena [4–6]. Considering their dynamics, in either dissipative or non-dissipative regimes [7–13], an analysis of scaling arguments and statistical properties yields new approaches, new formalisms, therefore moving forward the progress of the nonlinear science.

Our main goal in this paper is to introduce the dynamics of a system where a single particle (or an ensemble of non-interacting particles), suffers elastic impacts inside a closed domain bordered by two heavy periodically moving walls. The motivation for this model backs to Ulam [14] and Fermi [15], who developed a prototype model in order to explain the high energy of the cosmic rays, known as Fermi–Ulam model (FUM) [12], where an impact mechanism was setup, considering a particle suffering collision between two walls, where one is fixed and the other is periodically vibrating. Such a moving boundary system can be considered as billiard-like dynamical system [2,16,17]. Applications in different areas of research can be found such as: granular materials [18,19], microwaves [20], quantum dots [21,22], synchronization [23], mechanical vibrations [24,25], laser dynamics [26], chaos control [27,28], astrophysics [29], atom-optics [30,31], quantum effects [32,33], experimental devices [34,35], among many others.

This dynamical system that we aim to introduce in this paper differs from the original FUM [36–39]. Here, we are considering that the particle bounces between two heavy periodically vibrating walls, where each one of them has its own independent frequency of oscillation and amplitude of perturbation. Indeed, the kind of dynamics that we are proposing may be interpreted as a modeling for some known physical applications, such as ratchet-like dynamics [40–42], where the connection with our modeling comes from an interaction of the particle with two driven periodic oscillators, i.e., both vibrating walls. Another examples are: the photonic laser thruster, where a laser beam suffers successive reflections among moving mirrors [43–45], as well as classic and quantum δ-kicked rotators [46–50].

The dynamics of our proposed impact system is described by an original, nonlinear and area preserving mapping, which has four nonlinear terms and three control parameters and makes the mapping unique among similar impact dynamical systems present in the literature. The phase space presents mixed properties, with chaotic seas, invariant curves and KAM islands, where the ratio between frequencies of oscillation, influences the number of islands

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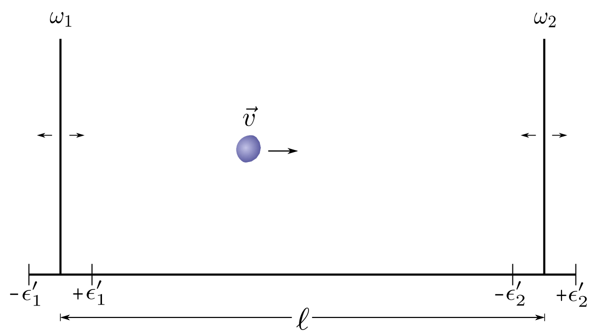

Fig. 1. Illustration of the dynamics of the impact system and its variables.

and the size of the chaotic sea. Because of this last property, the system denotes potential for further investigations concerning transport analysis, stickiness influence and anomalous diffusion [7,8,11,12]. Also, if one could introduce dissipation to the dynamics, we believe that crises between attractors [9] and self-similarity structures in the parameter space would be observed. So, in a primary study of our impact system, the behavior of the root mean square velocity was investigated, where an analytical description of its evolution was made. Our numerical simulations lead us to propose scaling arguments to describe the dynamics of the root mean square velocity. At the end, an universal collapse for these curves was obtained, which gives validity to our scaling hypothesis. Also, an analysis concerning the diffusion coefficient was made and shows a remarkable agreement with the analytical argument obtained by the investigation of the root mean square velocity as a function of the control parameters. Such agreement gives robustness to the modeling of our dynamical system.

The paper is organized as follows: in Sec. 2 we describe how the mapping was obtained and some dynamical and chaotic properties. Section 3 is devoted to discussing the behavior of the root mean square velocity via analytical investigation and numerical simulations. It also describes the scaling invariance of the root mean square velocity curves regarding the control parameters. Yet, an analysis of the diffusion coefficient also confirms the validity of the analytical investigations and of the scaling arguments. Finally in Sec. 4, we present our final remarks and conclusions.

2. The model and the mapping

The dynamical system under study in the paper basically consists of a particle of mass m moving in straight line trajectories, confined in a region of length ℓ, where in each boundary of this confined region, lies two heavy periodically moving walls. The velocity of the particle is said to be constant during the "flight time". It only changes, when the particle interacts with the moving walls. Fig. 1 displays a schematic illustration of the dynamical system and its variables.

In both heavy moving walls, the periodicity is given by a cosine function. Their equations of motion are $x_{w1} = \epsilon_1' \cos(\omega_1 t)$ for the left hand-side wall, said to be wall-1, and $x_{w2} = \epsilon_2' \cos(\omega_2 t)$ for the right hand side wall, said to be wall-2. Here, ϵ_1', ϵ_2', ω_1 and ω_2 are the respectively amplitudes of motion and frequencies of oscillation.

The collision scenario will be setup considering the static wall approximation (SWA) [36,37], where no transcendental equations must be solved to find the exact collision time. In this approximation both walls are said to be fixed (at rest), but the particle exchanges momentum and energy with the walls at each collision as if they were moving normally [51,52]. It had been shown before that in similar impact dynamical systems, the difference between the real dynamics and the SWA is nearly null, if one considers the whole accessible phase space [8,51].

The mapping dynamics will be described in the variables velocity v and time t. As an initial condition, we considered that the particle belongs to the wall-1, with an initial velocity v_0 with positive orientation and the initial time t_0. Once we decided to choose wall-1 as an initial condition (one could also start with wall-2, without any loss of generality), the mapping dynamics will be updated each time the particle collides again with wall-1.

Since we are considering static wall approximation, the time elapsed until the collision with the wall-2 can be set as $t_{1,2} = t_0 + \ell/v_0$. In the same manner, the return time will be $t_{2,1} = -\ell/v_1$, where now v_1 is the velocity of the particle after colliding with wall-2 and it has negative orientation. So the total time for the particle to leave wall-1, collide with wall-2, and come back to collide with wall-1 again is $t_t = t_{1,2} + t_{2,1}$, which gives us

$$t_t = t_0 + \ell \left[\frac{v_1 - v_0}{v_0 v_1} \right] . \tag{1}$$

Let us now obtain the expression for v_1. To evaluate the exchange of momentum and energy at each collision, we must consider a change in the reference frame, from inertial to non-inertial. So, one may consider $\vec{X}(t_0) = \vec{x_p}'(t_0) + \vec{x_{w2}}(t_{1,2})$, where $\vec{x_p}'$ is the position of the particle in the non-inertial reference frame, $\vec{x_{w2}}$ and $\vec{X}$ are respectively the equation of the wall-2 and the particle position in the inertial reference frame. After a time derivative we found $\vec{V}(t_0) = \vec{v_p}'(t_0) + \vec{v_{w2}}(t_{1,2})$. The term $\vec{v_p}'(t_0)$, is setup before the collision with wall-2 happens.

After the collision, and considering the conservation of momentum, one obtain $\vec{v_p}'(t_{1,2}) = -\vec{v_p}'(t_0)$. Coming back to the inertial reference frame, one may find $\vec{V}(t_{1,2}) = \vec{v_p}'(t_{1,2}) + \vec{v_{w2}}(t_{1,2})$. Rearranging properly the terms and setting $\vec{V}(t_{1,2}) = v_1$, $\vec{V}(t_0) = v_0$ and $\vec{x_{w2}}(t_{1,2}) = -\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2})$, one can finally obtain

$$v_1 = -v_0 - 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2}) . \tag{2}$$

Coming back to the total flight time expression set in Eq. (1), and replacing the expression for v_1 obtained in Eq. (2), one may find

$$t_t = t_0 + \ell \left[\frac{-v_0 - 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2}) - v_0}{v_0(-v_0 - 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2}))} \right] .$$

Rearranging the terms from the above expression, one may obtain

$$t_t = t_0 + 2\ell \left[\frac{1 + \frac{\epsilon_2' \omega_2}{v_0} \sin(\omega_2 t_{1,2})}{v_0 + 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2})} \right] . \tag{3}$$

Since we are evaluating the dynamics considering a total flight time from wall-1, until the particle collides again with it, we should set the velocity v_2 when the particle returns to the wall-1. So, applying the same procedure as done for v_1 one may obtain

$$v_2 = -v_1 - 2\epsilon_1' \omega_1 \sin(\omega_1 t_t) . \tag{4}$$

Replacing Eq. (2) into Eq. (4) and properly naming the terms: $v_0 = v_n$, $v_2 = v_{n+1}$, $t_t = t_{n+1}$ and $t_0 = t_n$, one may find

$$T : \begin{cases} v_{n+1} = v_n + 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2}) - 2\epsilon_1' \omega_1 \sin(\omega_1 t_t) \\ t_{n+1} = t_n + 2\ell \left[\frac{1 + \frac{\epsilon_2' \omega_2}{v_n} \sin(\omega_2 t_{1,2})}{v_n + 2\epsilon_2' \omega_2 \sin(\omega_2 t_{1,2})} \right] \end{cases} . \tag{5}$$

One can realize that in Eq. (5) there are too many control parameters in the mapping, ϵ_1', ϵ_2', ω_1, ω_2 and ℓ. In order to reduce these control parameters, let us set some dimensionless parameters as $\epsilon_1 = \epsilon_1'/\ell$, $\epsilon_2 = \epsilon_2'/\ell$, and $\tilde{\omega} = \omega_2/\omega_1$. Also, since we are considering the mapping update when the particle reaches wall-1, let us set a dimensionless velocity as $V_n = v_n/\omega_1 \ell$ and measure