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Formation of whispering gallery modes by scattering of an electromagnetic plane wave by two cylinders

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ABSTRACT

We report the effect of scattering of electromagnetic plane waves by two cylinders on whispering gallery mode (WGM) formation in a cylinder. WGM can occur because of the presence of additional cylinder scatterers at specific location, while WGMs can only form in a single cylinder for specific cylinder radius and/or wavelength values, the matching accuracy required would be much greater than that required in our model for the additional cylinders locations. Analysis of the general solution to the problem showed that the effect can be explained by the interference of waves scattered by additional cylinders and incident on the main cylinder.

1. Introduction

The term "whispering gallery modes" (WGMs) was introduced by Lord Rayleigh to explain the effects of sound propagation a circular gallery [1]. The name itself reflects the fact that sound in enclosed spaces can propagate along such concave walls. Laser radiation that is attached to the perimeter of a miniature disk by multiple reflections from concave walls can be treated as an optical analogue of the "whispering gallery". If the reflection surface is sufficiently smooth, the radiation then propagates with minimal losses. However, because the lateral surface is not smooth and has a nonzero radius of curvature, part of the wave then leaks out. When the radius of curvature increases, more radiation remains inside the disk. WGMs can thus be used to create electromagnetic cavities, as initially noted by R. Richtmyer [2]. This spherical form is the simplest form that can be used in fabrication of a resonator using WGMs. The interaction of spherical particles with electromagnetic waves has been studied theoretically for more than a century, dating back to the work of Mi [3], who considered the scattering of light by spherical particles, and Debye, who studied the scattering of waves on a sphere in the form of a series of refracted and reflected waves of various order [4]. However, despite the fact that this problem (the scattering of waves on an axisymmetric particle) is well known and has been studied for a long time, new and important results were obtained in 2004: the authors of Ref. [5] found and studied a narrow, high-intensity beam of light (called a photonic nanojet) that was generated at the shadow-side surfaces of dielectric cylinders that were illuminated by a plane wave. Photonic nanojet could also be formed using spheres [6], two-layer dielectric microsphere cylinders [7], and discs [8]. The influence of the incident light wave's polarization was investigated in [9]. The formation and transport processes of photonic nanojets produced by multiple cylinder scattering were calculated in Ref. [10]. The renewed interest to cylinder scattering of plane waves led to a detailed study of the conditions required to produce a WGM. V.V. Kotlyar et al. [11] found that the WGM formed a focal spot outside the cylinder, and also determined the contributions of the cylinder eigenmodes to WGM formation. The excitation of a whispering gallery resonator by a surface wave guided in a dielectric slab is analyzed in [12]. In general, fundaments of WGM propagation and its applications described in Ref. [13].

The effect of multiple cylinders scattering of an electromagnetic plane-wave on the formation of high field intensity areas studied in [14].

2. Modeling and results

Our model consists of two cylinders. One of these (marked as *A* on Fig. 1) is a basic cylinder, within which we consider WGM formation. The other cylinder (marked *B* on Fig. 1) has an assistive

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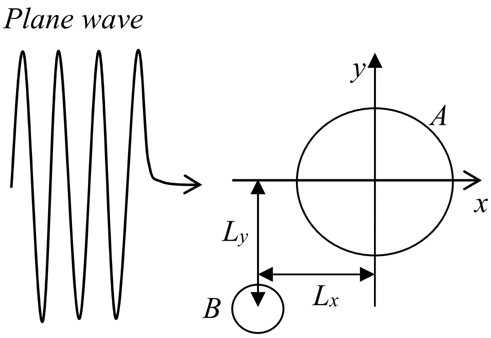

Fig. 1. Geometry to simulate scattering of plane wave by two cylinders.

function. The position of cylinder B is varied relative to the basic cylinder with the purpose of finding a location at which the intensity of the WGM increases. Our model, including all distances and notations, is presented in Fig. 1. The plane wave is incident from the left side on the pair of cylinders. The wave propagation direction is along the axis of symmetry of the cylinders, and is selected as the x axis. The origin of coordinates corresponds to the center of main cylinder A. The propagation and scattering of the electromagnetic plane wave was studied using MATLAB toolbox that was developed in Ref. [15]. A transverse-electric (TE) polarized plane wave ($\lambda = 532$ nm) was used as the incident wave. The mesh grid size in the space was equal to 0.04 µm (0.075 of wavelength). The electric permittivity $\varepsilon = 1.59$ (quartz glass), and cylinder's A radius $R_A = 4\lambda$. For cylinder B (where the latter is introduced below), the radii $R_B = 0.25R_A$ were used. All distances below are measured in µm.

The calculated distribution of the absolute field intensity value is shown on Fig. 2(a). As shown, the scattering process leads to the formation of a photonic jet for the single cylinder A. Next we used additional cylinder B. An image for comparison of the single cylinder case with that of our model is shown in Fig. 2(b) ($L_x = -2.7, L_y = 0.5$). Additionally, Fig. 3 shows absolute value of the total field as function of a distance between the center and edge of the disk.

To determine how the positions of cylinder B affect the maximal absolute field value inside the cylinder A, we varied its positions by moving the centers within the ring defined in polar coordinates as interval of radii $[R_A + R_B, R_A + 1.6 * R_B]$. Each step in the ring was $0.06R_B/10$ along the radius, with an angle step of $(\pi/2)/75$. The resulting picture is shown in Fig. 4. The positions of cylinder B that were used in Fig. 2 correspond with the data used for Fig. 4.

3. Formation of WGM due to scattering on neighboring cylinder

We associate the near-surface area of high field intensity with WGM propagation, because the specific feature of WGMs is that the high intensity field in these modes is concentrated near the cavity walls (Fig. 2(b), Fig. 3). The absence of high field intensity areas inside a single scattering cylinder A indicates that WGM is caused by the presence of the additional cylinder B. Also, from Fig. 2(b) we can conclude that in our case we have deal with mode $m = 33$ and such a high number is typical for WGM.

In general, the WGM are characterized by the specific value of the following relationship: cylinder radius/wavelength. As an example we consider simple expression $2\pi Rn/\lambda = T_{ml}$ [16], where T_{ml} is the lth root of the mth order Bessel function. This means that if the wavelength is known, then to determine the propagation of (ml) mode we must choose the cylinder radius based on the expression above. Additionally, when this mode makes a larger contribution to the field intensity, then the radius must be defined more accurately [13]. For example, for a mode with $m = 15$, when the intensity is increased by eight times, then the accuracy of the matching radius is $10^{-4}\lambda$ [14]. It therefore follows from Fig. 4 that to attain the same increase in intensity, it is sufficient to determine the location within accuracy of a few percent of the wavelength value.

In the case of two or more cylinders, the equation for the derivation the WGM will contain a contribution caused by the presence of satellite cylinder. To demonstrate this we use the fact that the eigenmodes inside a cylinder can be expanded into a series in the special functions [10, as instance]. The use of the boundary conditions leads to a system of linear equations for unknown expansion coefficients: $\mathrm{M} * \mathrm{A} = \mathrm{C}$, where A is a column of unknown coefficients, and the elements of matrix M and column C are determined by the known parameters. If cylinder is not illuminated by light, i.e. $\mathrm{C} = 0$, then system $\mathrm{M} * \mathrm{A} = 0$ have nontrivial solutions under a condition: $\mathrm{Det(M)} = 0$. This equation is known as characteristic, and it determines the eigenmodes of the cylinder, which characterized by an integer index m. The equation is valid for any number of the cylinders. The difference will be in the corresponding expressions for the determinant. Thus, eigenmodes will not be the same for the cases, for instance, of single and two cylinders. First of all, this conclusion means that for the case of two cylinders WGM are derived by many parameters (radii of the satellite cylinders, distance between cylinders, their mutual orientation, dielectric permittivity), but not only by a relation R/λ, as it was for single. Next, due to the presence of satellite cylinder WGM are formed by the interference of incident waves and waves scattered by additional cylinder. The latter also follows from the general formulas describing multiple cylinders scattering.

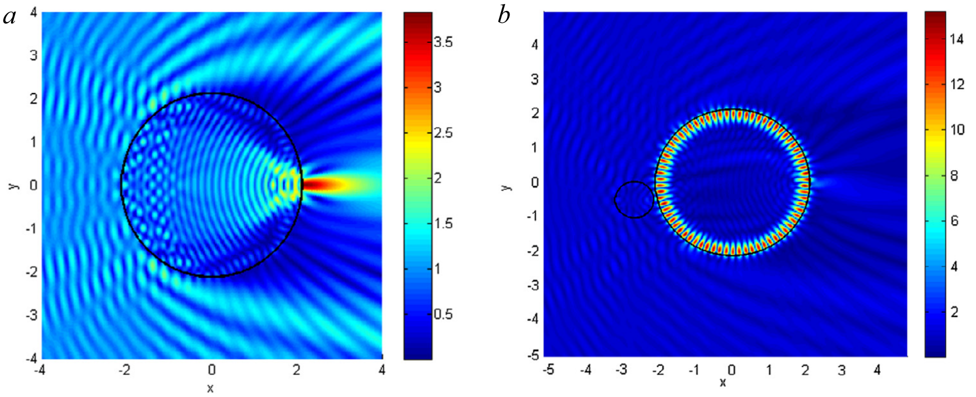

Fig. 2. Distribution of the absolute value of total field scattered by one (a) and two (b) dielectric cylinders. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)