Physics Letters A 381 (2017) 19–23

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Adopting epidemic model to optimize medication and surgical intervention of excess weight

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ARTICLE INFO

Article history:
Received 2 August 2016
Received in revised form 17 October 2016
Accepted 18 October 2016
Available online 21 October 2016
Communicated by C.R. Doering

Keywords:
Ordinary differential equation
Epidemic model
Optimization
Cost minimization
Weight

ABSTRACT

We combined an epidemic model with an objective function to minimize the weighted sum of people with excess weight and the cost of a medication and surgical intervention in the population. The epidemic model is consisted of ordinary differential equations to describe three subpopulation groups based on weight. We introduced an intervention using medication and surgery to deal with excess weight. An objective function is constructed taking into consideration the cost of the intervention as well as the weight distribution of the population. Using empirical data, we show that fixed participation rate reduces the size of obese population but increases the size for overweight. An optimal participation rate exists and decreases with respect to time. Both theoretical analysis and empirical example confirm the existence of an optimal participation rate, u^*. Under u^*, the weighted sum of overweight (S) and obese (O) population as well as the cost of the program is minimized. This article highlights the existence of an optimal participation rate that minimizes the number of people with excess weight and the cost of the intervention. The time-varying optimal participation rate could contribute to designing future public health interventions of excess weight.

1. Introduction

Excess weight has become a serious public health problem because of its prevalence and potential hazard. Based on a recent study from the Centers for Disease Control and Prevention (CDC), 69% of the adults in the US are overweight and more than 35% of all adults are obese [1]. In June 2013, the American Medical Association (AMA) officially classified obesity as a disease [2]. According to a study by Thomas et al., obesity prevalence is predicted to plateau independent of current preventative strategies [3]. This disease is not unique to the US. Nearly all developed countries and many developing ones are witnessing the problem of rapid growth of the overweight population [4]. Excess weight also affects different age groups. Studies have shown that young adults as well as the elderly face the same issue of weight control [5–7]. Furthermore, some scholars suggest that excess weight has a differential impact on health across subpopulations [8]. Excess weight results from an imbalance of energy [9]. When a person inputs more energy than he/she expends, that person stores the extra energy in the body thus increases the body weight. Excess weight can be further divided into two categories: overweight and obesity. According to the World Health Organization (WHO), overweight is defined as a Body Mass Index (BMI) greater than or equal to 25 and obesity is defined by a BMI greater than or equal to 30 [10]. The prevalence of excess weight is high when we use BMI as the standard measurement [11], but even higher when using other measures such as waist circumference [12]. The health consequences of excess weight include asthma, diabetes, hypertension and coronary atherosclerosis etc. [13–15]. Furthermore, obesity may exacerbate other diseases such as respiratory illnesses or infectious diseases [16,17]. Recently, overweight, as well as obesity, has been recognized as a social epidemic because it can pass from one person to another via peer pressure or social contact [18,19]. Studies show that people tend to spend time with those of similar weight. Contact with obese people influences individual's perception of obesity and living habits [20]. As a result, excess weight spreads similarly to a contagious disease. In order to help people lose weight, doctors and dietitians have adopted different methods to reduce the gap between energy intake and expenditure. Behavior changes such as increasing the amount of physical activity, improving diets and adjusting living habits are typical interventions to treat overweight and obesity [21–25]. However, for some people, particularly those who are severely overweight, these treatments may not be enough. Studies reveal that medication and surgical approaches are more effective in dealing with obesity for some specific groups [13, 26]. In this letter, we divide the adult population into three subgroups: N for normal weight, S for overweight and O for obese.

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http://dx.doi.org/10.1016/j.physleta.2016.10.032

Then we assume that excess weight (overweight and obesity) is a socially transmitted epidemic and adopt an epidemic model to study the behaviors of the population. Similar to the SIR modeling approach in epidemiology and references therein [27–30], the NSO model is consisted of ordinary differential equations describing the dynamics to each subgroup. More importantly, we introduce an intervention using medication and surgery treatment in the model with optimization [31]. This program aims at helping obese people lose weight. Through changing the participation rate u, we find an optimal u that minimizes the weighted sum of S, O and the cost of the intervention. This intervention offers public health policy suggestions at improving the health status of the population and containing cost simultaneously. To the best of our knowledge, few previous literatures have studied the issue of excess weight using compartmental model with optimization. We also present a real life example using data from the work of Santonja et al. to check the effectiveness of this intervention in real life [18].

2. Methods

We construct a compartmental model by dividing the adult population into three groups at time t: normal weight $N(t)$, overweight $S(t)$ and those who are obese $O(t)$. We define these groups based on different BMI values at time t: $N(t)$ for BMI < 25; $S(t)$ for $25 \le$ BMI < 30; and $O(t)$ for BMI > 30.

Based on previous research [18,27–30], we make a few assumptions about the population. First, we assume the population to be homogeneous, that is, individuals can contact with one another freely. Next, we assume that $N(t) + S(t) + O(t) = 1$ at any time t without loss of generality, implying an equal size of birth and death rate of the population. The conservation of the model is ensured. Then we assume that the average stay time of a person in adulthood is T weeks and N, S, O are uniformly distributed over time T [18]. We can write $\frac{1}{T}$ as the birth and death rate for each group. In addition, we consider excess weight to be a socially transmitted disease via human contact with a transmission rate.

For $\beta, \rho, \gamma, \epsilon, u \in [0, 1]$, the model reflecting the dynamics to each population group is as follows:

$$\begin{cases} \dot{N} = \frac{1}{T}N_0 - \frac{1}{T}N - \beta N(S+O) + \rho S, \\ \dot{S} = \frac{1}{T}S_0 - \frac{1}{T}S + \beta N(S+O) - \rho S - \gamma S + \epsilon(1-u)O + uO, \\ \dot{O} = \frac{1}{T}O_0 - \frac{1}{T}O - uO + \gamma S - \epsilon(1-u)O, \end{cases} \tag{1}$$

where, $\frac{1}{T}N_0$, $\frac{1}{T}S_0$ and $\frac{1}{T}O_0$ denote the number of people entering each group respectively as new adults while $\frac{1}{T}N$, $\frac{1}{T}S$ and $\frac{1}{T}O$ stand for the number of people leaving each group at time t respectively. $\beta N(S+O)$ represents the number of people who are infected and become overweight. These people, whose weight are normal originally, become overweight through social contact with overweight and obese people by revising their diets or living habits. There also exists a group of overweight people, ρS, who lose enough weight to become part of the normal weight. They are able to lose weight through improving diets or consuming more energy via physical exercises. But not everyone is able to change their lifestyle and some overweight people continue to gain weight. Eventually these people become overweight and they are represented by γS.

Since we introduced the intervention to help reduce weight for the obese population, there are two ways in the model they lose weight to become overweight. Similar to other subpopulation groups, obese people can switch from O to S is via changing behaviors such as diet and the amount of physical activity. Here $\epsilon(1-u)O$ people succeed in this manner. In addition, uO stands

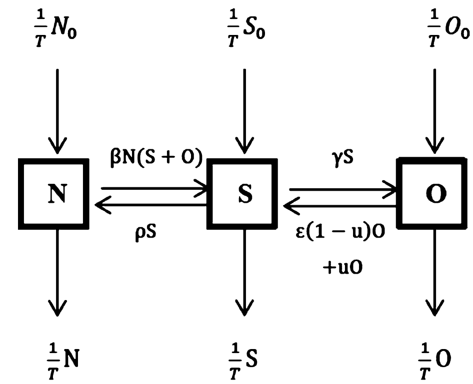

Fig. 1. Flow diagram of the mathematical model (1). Here boxes stand for subgroups of the population and arrows represent the transitions among subgroups with direction and corresponding parameter in the model.

for those who choose to take medication or go through surgery to lose weight. Scholars have reached some level of agreement that medication and surgery work better for some obese people, especially those who are severely obese and may need a fast way to lose excess weight [10].

Fig. 1 is a visualization of the model in which the population stock-flow is divided based on weight. The arrows between subpopulation groups indicate the possible methods people can switch category by behavior changes or via the medication and surgical intervention.

We consider two factors when evaluating the intervention: 1) the weighted number of overweight and obese people in the population; 2) the total cost of the intervention. These two goals measure the effectiveness of the intervention and contain the corresponding cost simultaneously. For the program to function at an optimal level, we need to minimize the weighted number of people with excess weight and keep the cost of running the program the lowest. The cost of the program, $\frac{u^2}{2}$, is related to the number of people taking medication and surgery to lose weight. We construct an objective function $V = \int_{t_0}^{\infty}(B_1 O + B_2 S + \frac{c}{2}u^2)\,dt$. B_1, B_2 and c stand for the weight of S, O and $\frac{u^2}{2}$ respectively. Thus the objective function V takes the weighted sum of overweight and obese people into account as well as the cost of medication and surgery. Then we are able to write the above problem in the following mathematical form:

$$\min_{0 \le u \le 1} \int_{t_0}^{\infty} (B_1 O + B_2 S + \frac{c}{2}u^2)\,dt,$$

subject to:

$$\begin{cases} \dot{N} = \frac{1}{T}N_0 - \frac{1}{T}N - \beta N(S+O) + \rho S, \\ \dot{S} = \frac{1}{T}S_0 - \frac{1}{T}S + \beta N(S+O) - \rho S - \gamma S + \epsilon(1-u)O + uO, \\ \dot{O} = \frac{1}{T}O_0 - \frac{1}{T}O - uO + \gamma S - \epsilon(1-u)O. \end{cases}$$

The corresponding Hamiltonian to the optimization problem above is:

$$\begin{aligned} H = {} & B_1 O + B_2 S + \frac{c}{2}u^2 + \lambda_1(\frac{1}{T}N_0 - \frac{1}{T}N - \beta N(1-N) + \rho S) \\ & + \lambda_2(\frac{1}{T}S_0 - \frac{1}{T}S + \beta N(1-N) - \rho S - \gamma S \\ & + \epsilon(1-u)O + uO) \\ & + \lambda_3(\frac{1}{T}O_0 - \frac{1}{T}O - uO + \gamma S - \epsilon(1-u)O). \end{aligned}$$