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Irreversibility analysis of hydromagnetic flow of couple stress fluid with radiative heat in a channel filled with a porous medium

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ABSTRACT

Numerical analysis of the intrinsic irreversibility of a mixed convection hydromagnetic flow of an electrically conducting couple stress fluid through upright channel filled with a saturated porous medium and radiative heat transfer was carried out. The thermodynamics first and second laws were employed to examine the problem. We obtained the dimensionless nonlinear differential equations and solves numerically with shooting procedure joined with a fourth order Runge-Kutta-Fehlberg integration scheme. The temperature and velocity obtained, used to analyse the entropy generation rate together with some various physical parameters of the flow. Our results are presented graphically and talk over.

Introduction

In industrial and engineering applications, the conductive couple stress fluids is vital and useful.

The rheological features of such fluids are vital in the extraction of crude oil from petroleum products, aerodynamics heating, electrostatic precipitation, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants, colloidal and suspension solutions. Stoke [1], proposed the micro-continuum theory of couple stress fluid with polar effects and defines the rotational field in terms of the velocity field for setting up the constitutive relationship between the stress and strain rate. Putting applications in consideration, Bujurke and Naduvinamani [2] studied the performance of narrow porous journal bearing lubricated with couple stress fluid. Lin [3] inspected the couple stress fluid model for squeeze film characteristics of finite journal bearings. The analytical solution for Hall and Ion-slip effects on mixed convection flow of couple stress fluid between parallel disks was conveyed by Srinivasacharya and Kaladhar [4]. Meanwhile, conductive couple stress fluids can support magnetic fields. The forces that act on the fluid is due to the presence of magnetic, thereby possibly altering the geometry and strength of the magnetic fields themselves. The relative strength of the advecting motions in the fluid which form the central point of magnetohydrodynamics (MHD) theory is the key issue for a particular conducting fluid. The heat transfer to magnetohydrodynamics non-Newtonian couple stress pulsatile flow between two parallel porous plates was studied by Adesanya and Makinde [5]. Muthuraj et al. [6] investigated numerically the combined effects of heat and mass transfer on MHD flow of a couple stress fluid in a horizontal wavy walled channel filled with a porous medium in the presence viscous dissipation. The hypothetical studied of MHD oscillatory slip flow and heat transfer in a channel filled with porous media was analysed by Adesanya and Makinde [7]. When the flow systems operate at high temperature, there exists thermal radiation effect. Heat transfer by concurrent radiation and convection are frequently encountered in numerous technological problems including combustion, furnace design, the design of high temperature gas cooled in nuclear reactors, nuclear reactor safety, fluidized bed heat exchanger, solar fans, solar collectors, natural convection in cavities, and many others. The combined radiation and mixed convection from a vertical wall with injection/suction in a non- Darcy porous medium was studied by Murthy et al. [8]. The impact of thermal radiation on free convection flow through a porous medium was investigated by Raptis [9]. Makinde and Animasaun [10] numerically considered the effects of nonlinear thermal radiation on MHD bioconvection of conducting nanofluid with quartic autocatalysis chemical reaction past an upper surface of a paraboloid of revolution.

Meanwhile, fluid flow and heat transfer processes are basically irreversible due to entropy production. Bejan [11] announced the concept of entropy generation analysis due to fluid flow and heat

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transfer as an effective tool to evaluate the performance of engineering devices. After his pioneering work, several researchers have analysed the fluid flow irreversibility problems under various physical situation [12–15]. Makinde and Eegunjobi [16] investigated the entropy generation rate in a couple stress fluid flows through a vertical channel filled with saturated porous media. Tasnim et al. [17] studied the effects of magnetic field on entropy generation rate in an isothermal porous two dimensional channel.

To the best of our knowledge, the mutual effects of magnetic field, thermal radiation, buoyancy force and convective heat transfer on entropy generation in a mixed convective flow of an electrically conducting couple stress fluid through a vertical channel packed with a saturated porous medium has not been reported yet in the literature. Our main objective is to tackle this problem theoretically by considering the inherent irreversibility in a mixed convection hydromagnetic flow of an electrically conducting couple stress fluid through a vertical channel packed with a saturated porous medium with radiative heat transfer. In the subsequent sections, the problem is formulated, we dimensionless the equations and solved. Relevant results are presented graphically and discussed.

Mathematical formulation

An electrically conducting incompressible, radiating couple stress fluid of an hydromagnetic steady flow in a vertical position channel, filled with a saturated homogeneous porous medium together with permeable walls as shown in Fig. 1 was considered. We assumed that the left wall (where fluid injection takes place) is upheld at a uniform temperature while the convective heat exchange with the surrounding fluid occurs at the right wall (where fluid suction occurs). The flow occurs in the direction of x-axis and the y-axis is taken perpendicular to it. The influences of an external, transversely applied, uniform magnetic fields of strength B_0 are on the flow field. The effect of magnetic Reynolds number and the induced electric field are assumed to be minor and insignificant. We denoted the distance between two permeable walls of the channel by a and the length by L. we put into consideration the thermal radiation that takes place during flow process in the channel as well as the velocity slip at the right.

Using Brinkman–Forchheimer flow model, the governing equations are obtained from the balance of linear momentum, energy and volumetric entropy generation rate equations as [6,12–14,16]:

$$-V\frac{du}{dy} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \upsilon\frac{d^2u}{dy^2} - \frac{\delta}{\rho}\frac{d^4u}{dy^4} - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon u}{k_1} - \frac{cu^2}{\rho\sqrt{k_1}} + g\beta(T - T_w), \tag{1}$$

$$-V\frac{dT}{dy} = \frac{k}{\rho c_p}\frac{d^2T}{dy^2} + \frac{\upsilon}{c_p}\left(\frac{du}{dy}\right)^2 + \frac{\delta}{\rho c_p}\left(\frac{d^2u}{dy^2}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2 u^2}{\rho c_p} + \frac{\upsilon u^2}{k_1 c_p} + \frac{cu^3}{\rho c_p\sqrt{k_1}}, \tag{2}$$

$$E_G = \frac{k}{T_w^2}\left(1 + \frac{16\sigma^* T^3}{3k^* k}\right)\left(\frac{dT}{dy}\right)^2 + \frac{\mu}{T_w}\left(\frac{du}{dy}\right)^2 + \frac{\delta}{T_w}\left(\frac{d^2u}{dy^2}\right)^2 + \frac{\sigma B_0^2 u^2}{T_w} + \frac{\mu u^2}{T_w k_1} + \frac{cu^3}{T_w\sqrt{k_1}}. \tag{3}$$

The suitable boundary conditions for the fluid velocity and temperature are given as

$$u = \frac{d^2u}{dy^2} = 0, T = T_w = 0, \text{ at } y = 0, \tag{4}$$

$$u = \frac{d^2u}{dy^2} = 0, -k\frac{dT}{dy} = h(T - T_w), \text{ at } y = a, \tag{5}$$

here u is the axial velocity, h represents wall heat transfer coefficient, μ stands for the dynamic viscosity, ρ is the fluid density, E_G is the entropy generation rate, T is the fluid temperature, c_p is specific heat at constant pressure, σ is the electrical conductivity, g is the gravitational acceleration, δ is the fluid particle size effect due to couple stresses, V is the wall injection/suction velocity, T_w is the channel left wall temperature, β is the thermal expansion coefficient, k is the thermal conductivity of the fluid, $k_1$is the porous media permeability, c is the empirical constant in the second order (porous inertia) resistance such that $c = 0$ corresponds to the Darcy law. By assuming Rosseland approximation [8–10] the radiative heat flux is taken as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^* T^3}{3k^*}\frac{\partial T}{\partial y}, \tag{6}$$

where σ^* is the Stefan–Boltzmann constant and $k*$ is the mean absorption coefficient. We present the dimensionless variables and parameters as follows:

$$\begin{aligned} &\eta = \tfrac{y}{a}, X = \tfrac{x}{a}, \theta = \tfrac{T - T_w}{T_w}, \upsilon = \tfrac{\mu}{\rho}, w = \tfrac{ua}{\upsilon}, \Pr = \tfrac{\mu c_p}{k}, Ec = \tfrac{\upsilon^2}{c_p T_w a^2}, \\ &\lambda = \tfrac{\delta}{\rho\upsilon a^2}, A = -\tfrac{\partial\bar{P}}{\partial X}, M = \tfrac{\sigma B_0^2 a^2}{\rho\upsilon}, Nr = \tfrac{16\sigma^* T_w^3}{3k^* k}, Ns = \tfrac{E_G a^2}{k}, \\ &S_1 = \tfrac{a^2}{k_1}, S_2 = \tfrac{ca}{\rho\sqrt{k_1}}, \mathrm{Re} = \tfrac{Va}{\upsilon}, \bar{P} = \tfrac{a^2 p}{\rho\upsilon^2}, Gr = \tfrac{g\beta T_w a^3}{\upsilon^2}, Bi = \tfrac{ah}{k}. \end{aligned} \tag{7}$$

Replacing Eq. (7) into Eqs. (1)–(6), we get,

$$\frac{d^2w}{d\eta^2} - \lambda\frac{d^4w}{d\eta^4} + \mathrm{Re}\frac{dw}{d\eta} - (M + S_1)w - S_2w^2 + Gr\theta + A = 0, \tag{8}$$

$$\left[1 + Nr(\theta+1)^3\right]\tfrac{d^2\theta}{d\eta^2} + 3Nr(\theta+1)^2\left(\tfrac{d\theta}{d\eta}\right)^2 + \Pr\mathrm{Re}\tfrac{d\theta}{d\eta} + \Pr Ec\left[\left(\tfrac{dw}{d\eta}\right)^2 + \lambda\left(\tfrac{d^2w}{d\eta^2}\right)^2 + (M + S_1)w^2 + S_2w^3\right] = 0, \tag{9}$$

$$Ns = \left[1 + Nr(\theta+1)^3\right]\left(\frac{d\theta}{d\eta}\right)^2 + \Pr Ec\left[\left(\frac{dw}{d\eta}\right)^2 + \lambda\left(\frac{d^2w}{d\eta^2}\right)^2 + (M + S_1)w^2 + S_2w^3\right], \tag{10}$$

with

$$w = \frac{d^2w}{d\eta^2} = 0, \theta = 0, \text{ at } \eta = 0, \tag{11}$$

$$w = \frac{d^2w}{d\eta^2} = 0, \frac{d\theta}{d\eta} = -Bi\theta, \text{ at } \eta = 1, \tag{12}$$

where Pr is the Prandtl number, Re is the injection/suction Reynolds number, Gr is the Grashof number, Bi is the Biot number, λ is the couple stress parameter, Ec is the Eckert number, A is pressure gradient, S_1 is the porous medium shape factor parameter, S_2 is the second order porous medium resistance parameter, M is the magnetic field parameter, Br (=$EcPr$) is the Brinkmann number and Nr is the radiation parameter. Other quantities of concern are the skin friction coefficients (C_f), Nusselt number (Nu) and the Bejan number (Be) which are given as

$$C_f = \frac{\rho h^2 \tau_w}{\mu^2} = \left.\frac{dw}{d\eta}\right|_{\eta=0,1},$$
$$Nu = -\frac{hq_m}{kT_w} = -\left[1 + Nr(\theta+1)^3\right]\left.\frac{d\theta}{d\eta}\right|_{\eta=0,1}, Be = \frac{N_1}{Ns} = \frac{1}{1+\phi}, \tag{13}$$