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Mechanical analysis of Chen chaotic system

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ABSTRACT

The Chen chaotic system is transformed into Kolmogorov type system, which is decomposed into four types of torques: inertial torque, internal torque, dissipation and external torque. By the combinations of different torques, five cases are studied to discover key factors of chaos generation and the physical meaning. The conversion among Hamiltonian energy, kinetic energy and potential energy is investigated in these five cases. The relationship between the energies and the parameters is studied. It concludes that the combination of these four types of torques is necessary conditions to produce chaos, and any combination of three types of torques cannot produce chaos in Chen system.

1. Introduction

In 1963 the meteorologist E. N. Lorenz found chaos in a nonlinear differential equations, which is one of the first example of deterministic chaos in dissipative systems [1]. Chaos is abundant in nature and has been discovered in many branches of science, such as forced pendulum[2], nonlinear optical devices [3], classical many-body systems (three-body problem) and particle accelerators [4], chemical reactions [5], Fluids near the onset of turbulence [6], biological models for population dynamics [7].

In recent years, chaos research has achieved a great development in the generation and analysis of numerical chaotic system, circuit implementation, chaos-based cryptography application, etc. As regards chaos generation, many chaotic models have been constructed, such as Chen system [8], L*ü* system [9], Qi 3-D four-wing chaotic system [10]. Generally speaking, these systems were generated based on existing systems, for instance Lorenz system, Chua circuit or Rössler system [11]. They were constructed by route of increasing linear term or nonlinear terms or dimension and confirmed by numerical simulations [8–10,12,13]. The researchers have strived for new shapes or complex topological construction of chaotic attractors, such as multi-wing or multi-scroll [10,12].

To distinguish, a system is called physical chaotic system, if it is derived from physical process or sustained in physical background; otherwise, the system is called numerical chaotic system when it is made by numerical simulation. Most research of chaos focuses on the dynamical analysis of these numerical chaotic systems, even for the Lorenz system. The research topics usually are numerical calculation, aperiodic solution, sensitivity to initials, bifurcation theory, power spectrum, Lyapunov exponent, fractal dimension, system control and synchronization, etc.

Only a few researchers have investigated the mechanics and physical background of chaos [14,15], such as energy conservation, physical meanings and background, transformation among internal energy, dissipation and external force by far. Hence, it is an interesting and challenging problem to find the corresponding terms in these numerical chaotic systems. Mechanics deals with the dynamics of particles, rigid bodies, continuous media (fluid, plasma, and solid) [16,17], studies the transformation of different types of energy and forces and clarifies the physical meanings. Chaotic system is fairly comprehensive in terms of structure of terms, and interaction. If chaos is investigated in mechanics, much more fundamentals could be discovered. Qi and Liang [21] transformed the Qi four-wing chaotic system to Kolmogorov system, made force analysis and interpreted the state of chaos as angular momentum.

Arnold [18] presented a Kolmogorov system to describe dissipative-forced dynamical systems or hydro-dynamic instability with Hamiltonian function. Pasini and Pelino [15] investigated the Lorenz system and gave a unified view of Kolmogorov and Lorenz systems. Furthermore, employing the extended Kolmogorov system, Pelino, et al. [19] studied the energy cycle of Lorenz system. Therefore, Hamiltonian function and Kolmogorov system are a good start to study chaotic system in terms of mechanics.

The paper interprets the state variables of the Chen system as angular momentum, and decomposes the vector field into iner-

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tial torque, internal torque, dissipative torque and external torque. Different dynamical modes are proposed from the combination of different torques, and their dynamic and energy characteristic are studied to show the relationship between these torques and chaos generation.

In literatures, bifurcation is basic analysis of chaotic dynamics. As parameter changes, the system produces different modes, such as sink, periodic, multi-periodic, torus and chaos, etc. However, what is the parameter meaning? Furthermore, When the parameter changes to that range of value, why does the system generate chaos or other modes? This paper presents causes of different types of modes in terms of the physical meaning in mechanics.

The rest paper is organized as followings: Section 2 transforms Chen system into the Kolmogorov system. Section 3 analyzes the mechanism of different dynamic modes. A brief conclusion is made in Section 4.

2. Mechanics of Chen chaotic system

The Chen chaotic system in [8] is described as

$$\begin{cases}\dot{x}_1 = -ax_1 + ax_2 \\ \dot{x}_2 = -x_1x_3 - (c-a)x_1 + cx_2 \\ \dot{x}_3 = x_1x_2 - bx_3,\end{cases} \tag{1}$$

where a, b, c are positive real numbers, and $c < a < 2c$. Typically, when $a = 35, b = 3$ and $c = 28,$ the system has a unique chaotic attractor.

To discover the physical analogue of the state variables and mechanics of the system, we introduce Kolmogorov system in 3-D form as

$$\dot{x} = \{x, H\} - \Lambda x + f, \tag{2}$$

where $x = [x_1, x_2, x_3]^T$, and the antisymmetric brackets $\{\cdot,\cdot\}$ denotes the algebraic structure of the kinetic energy part of Hamiltonian function H, and a cosymplectic matrix J, or Lie-Poisson structure [17]

$$\{F, G\} = J_{ik}\partial_i F \partial_k G.$$

System (2) was originally by Kolmogorov to describe dissipative-forced dynamical systems or hydro-dynamic instability, as reported in [18]. The force (or torque), $\{x, H\}$, in Euler equation is called inertial force or centrifugal force. System (2) is the generalized Euler equation [17] with the dissipative force and the external force.

In [15], the authors discussed the relation between the Kolmogorov system and the well-known Lorenz equations. A transformation is applied to the Chen system,

$$y_1 = x_1,\ \ y_2 = x_2,\ \ y_3 = x_3 + c.$$

Then system (1) is rewritten as

$$\begin{cases}\dot{y}_1 = -ay_1 + ay_2 \\ \dot{y}_2 = -y_1y_3 - ay_1 + cy_2 \\ \dot{y}_3 = y_1y_2 - by_3 - bc,\end{cases} \tag{3}$$

Define the Hamiltonian $H = K + U$,where

$$K = \frac{1}{2}(y_1^2 + 2y_2^2 + 2y_3^2) \text{ and } U = ay_3.$$

Here, the Hamiltonian H includes the kinetic energy K and the potential U. Then, system (3) can be described as the Kolmogorov system,

$$\dot{y} = \begin{pmatrix} ay_2 \\ -y_1y_3 - ay_1 \\ y_1y_2 \end{pmatrix} - \begin{pmatrix} ay_1 \\ -cy_2 \\ by_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -bc \end{pmatrix} \tag{4}$$
$$= \{y, H\} - \Lambda y + f,$$

where $y = [y_1, y_2, y_3]^T$, $\Lambda = diag\{a,\ -c,\ b\}$ and $f = (0,\ 0,\ -bc)^T$.

Remark 1. The variable x analogs the angular momentum, and the time derivative $\dot{y}$ denotes the reaction torque of rigid body or fluid flow. The first term $\{y, H\}$ is conserved term which includes inertial torque generated by kinetic energy and internal torque released by the potential. The second term Λy represents dissipative torque, which could be friction or heating exchange or viscous force. And the last term f is external torque.

3. Analysis and case study

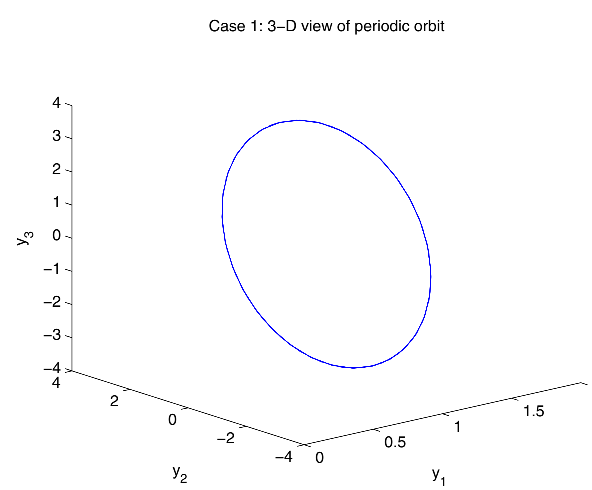
In this section, we investigate the effect of different types toques on system (4) to discover the key factors producing chaos.

Case 1: The system only contains the term of inertial torque (generated by kinetic energy K), that is,

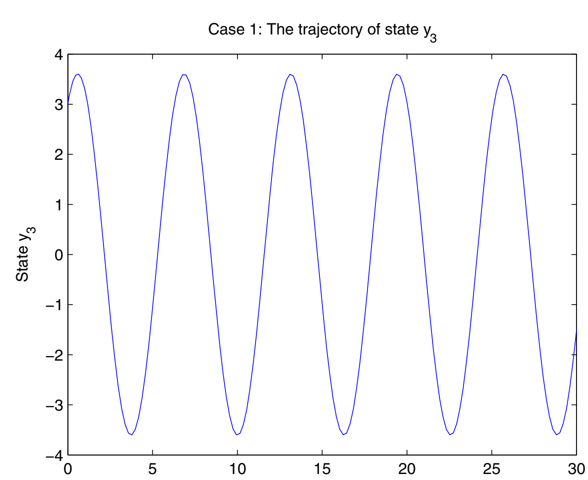
$$\dot{y} = \{y, K\} = \begin{pmatrix} 0 \\ -y_1y_3 \\ y_1y_2 \end{pmatrix}. \tag{5}$$

The derivative of kinetic function is

$$\dot{K} = y_1\dot{y}_1 + 2y_2\dot{y}_2 + 2y_3\dot{y}_3 = 0.$$



(a) 3-D view of the periodic orbit



(b) the trajectory of state y_3

Fig. 1. System is under inertial torque.