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Characteristics of the evolution of cooperation by the probabilistic peer-punishment based on the difference of payoff

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ABSTRACT

Regarding costly punishment of two types, especially peer-punishment is considered to decrease the average payoff of all players as well as pool-punishment does, and to facilitate the antisocial punishment as a result of natural selection. To solve those problems, the author has proposed the probabilistic peer-punishment based on the difference of payoff. In the limited condition, the proposed peer-punishment has shown the positive effects on the evolution of cooperation, and increased the average payoff of all players.

Based on those findings, this study exhibits the characteristics of the evolution of cooperation by the proposed peer-punishment. Those characteristics present the significant contribution to knowledge that for the evolution of cooperation, a limited number of players should cause severe damage to defectors at the large expense of their payoff when connections between them are sparse, whereas a greater number of players should share the responsibility to punish defectors at the relatively small expense of their payoff when connections between them are dense.

1. Introduction

Many studies discuss the effect of costly punishment that a player pays some cost, and punishes a defector on the emergence and the increase of cooperators (i.e. the evolution of cooperation). The previous studies [1–4] show that costly punishment enhances cooperation in group interactions among players (i.e. public goods games), whereas some studies question the significance of costly punishment for the evolution of cooperation. For example, the studies [5–7] propose the evolutionary puzzle of costly punishment, and also Dreber et al. [8] show that costly punishment increases the level of cooperation, whereas it decreases the average payoff. Rand and Nowak [9] point out that costly punishment facilitates the antisocial punishment like retaliation of a defector on a cooperator as a result of natural selection. On the other hand, Fowler [3] considers nonparticipants in voluntary public goods games, and shows that punishment can allow the evolution of any types of strategy. O' Gorman et al. [10] find that allowing a single player to punish increases cooperation to the same level as allowing each group member to punish, and results in greater group profits. The relaxation of both the fixed fine and the cost of punishment can explain both the spontaneous emergence of punishment and the prevention of the prevalence of defectors [11,12].

Note that there are two alternative notions regarding costly punishment, i.e. peer-punishment [1–14] and pool-punishment [15–17]. Peer-punishment applies direct face to face punishment, whereas pool-punishment is based on multi-point, collective interaction among group members. The previous paper [18] refers to the following relevant studies. As for peer-punishment in the public goods game, Helbing et al. [19] show that the consideration of punishment allows us to understand the establishment and spreading of cooperators who punish defectors. Szolnoki et al. [20] study the impact of pool-punishment in the spatial public goods game with cooperators, defectors, and pool-punishers as the three competing strategies. Helbing et al. [19,21,22] particularly discuss the efficiency of pool-punishment in maintaining socially advantageous states contrasted with that of peer-punishment. Chen et al. [23] show that in the public goods game, the introduction of punishment has a positive effect on cooperation especially for large group size, whereas an intermediate group size is not best for cooperation. Sasaki et al. [24] introduce the deposit that will be refunded as long as the committers faithfully cooperate in the donation game, and punish free riders and noncommitters. Perc [25] shows that pool-punishment in structured populations is sustainable, but only if second-order free-riders are sanctioned as well, and to such a degree that those free-riders cannot prevail. Those free-riders are eliminated by means of a discontinuous phase transition that shifts the evolution rather explosively in favor of the punishers.

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Regarding costly punishment of two types, this study discusses the evolution of cooperation by peer-punishment in the spatial prisoner's dilemma game of pairwise interactions. As described before, there is some criticism regarding peer-punishment [8,9]. In addition, the subject experiment of Wu et al. [26] shows that costly punishment does not always increase cooperation in pairwise interactions. To solve those problems, the previous study [18] has proposed the probabilistic peer-punishment based on the difference of payoff as follows. Firstly, every player punishes only defectors to prevent the antisocial punishment. Secondly, the degree of punishment dynamically changes based on the payoff of every punishing player, and the probability that he/she punishes others is directly proportional to the difference between his/her payoff and the payoff of others. In the limited condition, the proposed peer-punishment has shown the positive effects on the evolution of cooperation, and increased the average payoff of all players.

Based on those findings, this study exhibits the characteristics of the evolution of cooperation by the proposed peer-punishment, especially how the number of cooperators, the number of defectors, and the average payoff of all players increase or decrease depending on the increase of the coefficient of punishment that is the main control variable of the proposed peer-punishment. Those characteristics present the significant contribution to knowledge that for the evolution of cooperation, a limited number of players should cause severe damage to defectors at the large expense of their payoff when connections between them are sparse, whereas a greater number of players should share the responsibility to punish defectors at the relatively small expense of their payoff when connections between them are dense.

2. Model

The model of this study is similar to Nowak and May's basic framework of the spatial prisoner's dilemma game [27]. Every player has each strategy of two types, defection (=defector) and cooperation (=cooperator), matches the other players having connections with him/her, and then acquires the cumulative payoff from all matches. In this study, N is the number of all players, players i and j are two players of the match ($i \neq j$, $1 \leq i, j \leq N$), $s(i)$ and $s(j)$ are their strategy, and $P(i)$ and $P(j)$ are their payoff. By utilizing the payoff matrix A of Eq. (1), $P(i)$ can be expressed as the following Eq. (2). Note that $s(i)$ and $s(j)$ are either (1 0) (cooperator) or (0 1) (defector) of unit vectors. $O(i)$ denotes the set of the other players having connections with player i.

$$A = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} (1 < b \leq 2) \tag{1}$$

$$P(i) = \sum_{j \in O(i)} s(i) A s(j)^T \quad (i \neq j,\ 1 \leq i, j \leq N) \tag{2}$$

The parameters N, b, and the initial ratio of the number of defectors to the number of cooperators of this study follow those of the previous study [18]. That is, $N=1000$, $b=1.5$, and this initial ratio approximately equals one to one. Defectors and cooperators are randomly distributed in every simulation run. The spatial structure of connections of this study is the locally connected ring with periodic boundary conditions of Watts and Strogatz [28] of N players (vertices). Each vertex of the lattice exhibits each player, and the number of players having connections with player i is $k(i)$. The average of $k(i)$ ($<k>$) can be expressed as follows: $<k>= \frac{1}{N} \sum_{1 \leq i \leq N} k(i)$, and this study deals with three different cases of $<k>=4$, 8, and 16, respectively. The topology of connections defining the relationship of every player is three types, i.e. the regular [28], the (completely) random [28], and the scale-free known as the Barabási–Albert model [29] also following the previous study [18]. The detail of the construction of each type of the topology of connections is described in the Methods of the previous study [18]. Fig. 1 shows the sample initial state of the regular topology of connections of $<k>= 4$ [18]. Note that this figure has only 20 players in order to make clear the spatial structure of connections.

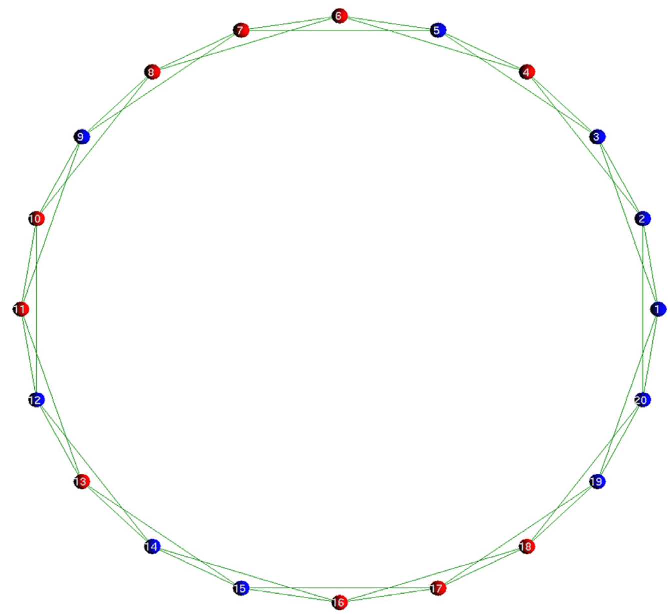

Fig. 1. This figure is the same as Fig. 1 of the previous paper [18], and shows the sample initial state of the regular topology of connections of $<k>=4$. The regular topology of connections means that the number of players having connections with player i ($=k(i)$) is the same regarding all players. The spatial structure of connections is defined as the one dimensional lattice of periodic boundary conditions, and each vertex of the lattice exhibits each player. Note that this figure has only twenty players ($N=20$) in order to make clear the spatial structure of connections. The ratio of the number of defectors (red) to the number of cooperators (blue) approximately equals one to one, and defectors and cooperators are randomly distributed in every simulation run. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

As described in the Introduction, this study exhibits the characteristics of the evolution of cooperation by the probabilistic peer-punishment based on the difference of payoff [18]. In the following formulation of the proposed peer-punishment, we utilize the number of players $j \in O(i)$ that satisfies both $P(j) > P(i)$ and $s(j)=(0\ 1)$ (defector) as $n(i)$, and r as the coefficient of punishment ($0 \leq r \leq 1$). As shown in the following Eqs. (3) and (4), when $P(i)(1-rn(i)) > 0$ and $P(i) < P(j) \leq 2P(i)$ hold, player i pays the cost $rP(i)$ and punishes player j of $s(j)=(0\ 1)$ by causing the damage $rP(i)$ to player j with the probability $q_i(j)$. When $P(i)(1-rn(i))> 0$ and $P(j) > 2P(i)$ hold, $q_i(j)$ equals 1. The decrease of payoff by punishing and punished is independently calculated regarding all players, and finally $P(i)'$ is set to 0 when it becomes a negative value. Therefore, $P(i)'$ and $P(j)'$ cannot be negative. Note that in the case of $r=0$ and 1, the proposed peer-punishment does not work because $rP(i)$ equals 0 in the case of $r=0$, and $P(i)(1-rn(i)) > 0$ never holds in the case of $r=1$ when $P(i) > 0$ and $n(i) \geq 1$ hold.

$$q_i(j) = \frac{P(j) - P(i)}{P(i)}, P(i) > 0 \tag{3}$$

$$P(i)' = P(i) - rP(i) \\ P(j)' = P(j) - rP(i) \tag{4}$$

After the payoff of all players changes due to all punishing and punished activities, as the following Eq. (5), player i chooses the strategy of player $j_{\max} \in i \cup O(i)$ for his/her strategy of the matches of the next generation. When two or more players have the max-