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A NOTE ON EINSTEIN FOUR-MANIFOLDS WITH POSITIVE CURVATURE

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Abstract. In this short note we prove that an oriented Einstein four-manifold with $\mathrm{Ric} = g$ and sectional curvature $K \geq \frac{1}{30}(19 - \sqrt{271})$ is isometric to (S^4, g_0) or $(\mathbb{C}P^2, g_{FS})$ up to rescaling.

1. Introduction

A Riemannian manifold $(M^n,\ g)$ is Einstein if its Ricci curvature is a multiple of the metric, i.e., $\mathrm{Ric} = \lambda g$ for some constant λ.

The classification of Einstein four-manifolds with positive sectional curvature is one of the basic problems in differential geometry. The first result goes back to Berger more than a half century ago, Berger [1] proved that Einstein four-manifolds with $\frac{1}{4}$-pinched sectional curvature are isometric to (S^4, g_0). Hitchin (Theorem 13.30 in [2]) proved that half conformally-flat Einstein four-manifolds with positive scalar curvature are isometric to either (S^4, g_0) or $(\mathbb{C}P^2, g_{FS})$ up to rescaling. Gursky and LeBrun [8] proved an interesting gap theorem for the L^2-norm of (anti-)self-dual Weyl curvature, using which they proved that Einstein four-manifolds with nonnegative sectional curvature and positive intersection form are isometric to $(\mathbb{C}P^2, g_{FS})$ up to rescaling. Yang [13] proved that oriented Einstein four-manifolds with $\mathrm{Ric} = g$ and sectional curvature $K \geq \frac{1}{120}(\sqrt{1249} - 23) \approx 0.102$ are isometric to (S^4, g_0) or $(\mathbb{C}P^2, g_{FS})$ up to rescaling. Costa [5] later relaxed Yang's condition to $K \geq \frac{1}{6}(2 - \sqrt{2}) \approx 0.097$.

In this short note, we further relax the condition on the sectional curvature, precisely we prove,

Theorem 1.1. *An oriented Einstein four-manifold with* $\mathrm{Ric} = g$ *and sectional curvature* $K \geq \frac{1}{30}(19 - \sqrt{271}) \approx 0.0846$ *is isometric to* (S^4, g_0) *or* $(\mathbb{C}P^2, g_{FS)}$ *up to rescaling.*

Remark 1.1. *The curvature in Theorem 1.1 can be further relaxed to* $K \geq 0.08433$*, see Remark 3.1.*

The rest of the paper is organized as follows. In Section 2, we recall the Weitzenböck formula and estimates for the (anti-)self-dual Weyl curvature. In Section 3, we prove Theorem 1 by combining the gap theorem of Gursky and LeBrun [8] and the argument of Yang [13].

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