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# A NOTE ON EINSTEIN FOUR-MANIFOLDS WITH POSITIVE CURVATURE

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ABSTRACT. In this short note we prove that an oriented Einstein four-manifold with  $\text{Ric} = g$  and sectional curvature  $K \geq \frac{1}{30}(19 - \sqrt{271})$  is isometric to  $(S^4, g_0)$  or  $(\mathbb{C}P^2, g_{FS})$  up to rescaling.

## 1. INTRODUCTION

A Riemannian manifold  $(M^n, g)$  is Einstein if its Ricci curvature is a multiple of the metric, i.e.,  $\text{Ric} = \lambda g$  for some constant  $\lambda$ .

The classification of Einstein four-manifolds with positive sectional curvature is one of the basic problems in differential geometry. The first result goes back to Berger more than a half century ago, Berger [1] proved that Einstein four-manifolds with  $\frac{1}{4}$ -pinched sectional curvature are isometric to  $(S^4, g_0)$ . Hitchin (Theorem 13.30 in [2]) proved that half conformally-flat Einstein four-manifolds with positive scalar curvature are isometric to either  $(S^4, g_0)$  or  $(\mathbb{C}P^2, g_{FS})$  up to rescaling. Gursky and LeBrun [8] proved an interesting gap theorem for the  $L^2$ -norm of (anti-)self-dual Weyl curvature, using which they proved that Einstein four-manifolds with nonnegative sectional curvature and positive intersection form are isometric to  $(\mathbb{C}P^2, g_{FS})$  up to rescaling. Yang [13] proved that oriented Einstein four-manifolds with  $\text{Ric} = g$  and sectional curvature  $K \geq \frac{1}{120}(\sqrt{1249} - 23) \approx 0.102$  are isometric to  $(S^4, g_0)$  or  $(\mathbb{C}P^2, g_{FS})$  up to rescaling. Costa [5] later relaxed Yang's condition to  $K \geq \frac{1}{6}(2 - \sqrt{2}) \approx 0.097$ .

In this short note, we further relax the condition on the sectional curvature, precisely we prove,

**Theorem 1.1.** *An oriented Einstein four-manifold with  $\text{Ric} = g$  and sectional curvature  $K \geq \frac{1}{30}(19 - \sqrt{271}) \approx 0.0846$  is isometric to  $(S^4, g_0)$  or  $(\mathbb{C}P^2, g_{FS})$  up to rescaling.*

**Remark 1.1.** *The curvature in Theorem 1.1 can be further relaxed to  $K \geq 0.08433$ , see Remark 3.1.*

The rest of the paper is organized as follows. In Section 2, we recall the Weitzenböck formula and estimates for the (anti-)self-dual Weyl curvature. In Section 3, we prove Theorem 1 by combining the gap theorem of Gursky and LeBrun [8] and the argument of Yang [13].

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