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Acceleration waves on random fields with fractal and Hurst effects

Vinesh V. Nishawala *, Martin Ostoja-Starzewski

Department of Mechanical Science and Engineering, Institute for Condensed Matter Theory and Beckman Institute, University of Illinois at Urbana–Champaign, Urbana, IL 61820, USA

HIGHLIGHTS

- Acceleration waves on multiscale random fields are evaluated.
- Random fields exhibit fractal and Hurst characteristics.
- Fields with high Hurst coefficient, show strongest deviation from homogeneous result.

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ABSTRACT

In this study, we determine the effect of spatial randomness on the probability of shock formulation and the distance to form shocks from acceleration waves as a function of the initial amplitude. The noise is applied to the dissipation and elastic nonlinearity of the system for two different cases: (i) two variables with the same noise of varying intensity and (ii) four variables with the same noise of varying intensity. The random fields used here are unique as they can capture and decouple the field's fractal dimension and Hurst parameter. We focus on determining the driving parameter, either fractal or Hurst, which is significant in altering the response of the system.

1. Introduction

The propagation of waves in heterogeneous media is a topic of interest especially for systems in nature, as they are seldom homogeneous. While other stochastic models exist [1,2], these models do not effectively capture fractal and Hurst characteristics which are found in nature [3,4]. Recently, stochastic models have been developed which capture and decouple the fractal and Hurst characteristics [5,6]. These models are known as Cauchy and Dagum random processes.

In this paper, we evaluate the effects of these random fields (RFs) on the amplitude of acceleration waves. The amplitude of acceleration waves is governed by the Bernoulli equation [7–9] which is of the form,

$$\frac{\mathrm{d}A(x)}{\mathrm{d}x} = -\mu A(x) + \lambda A(x)^2. \tag{1}$$

For the equation above, A denotes the jump in particle acceleration, x is position and μ and λ denote dissipation and elastic non-linearity, respectively. Due to competing effects of dissipation and non-linearity, there is a possibility of shock or caustic

* Corresponding author.
E-mail address: nishawa2@illinois.edu (V.V. Nishawala).

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formation at some finite distance x_∞. For homogeneous μ and λ, the analytical solution of (1) is

$$A(x) = \left(\frac{\lambda}{\mu} + \left(\frac{1}{A_o} - \frac{\lambda}{\mu}\right)\exp(\mu x)\right)^{-1}, \tag{2}$$

where A_o is the initial amplitude of the wave.

For homogeneous systems, a shock forms when the initial amplitude, A_o, is greater than the critical amplitude, A_c. In this study, we want to determine to what extent the fractal dimension, D, Hurst exponent, H, or some combination, is significant in altering the response. More specifically, we look at the effects on A_c and x_∞. For a homogeneous system, they are easily determined as

$$A_c = \frac{\mu}{\lambda}, \text{ and } x_\infty = -\frac{1}{\mu}\ln\left(1 - \frac{\mu}{\lambda A_o}\right). \tag{3}$$

While the RFs of Cauchy and Dagum type are wide-sense stationary, given that their spectral densities are not known in explicit forms, an analytical approach (using stochastic differential equations) is not possible. Thus, we have to resort to a Monte Carlo approach: generate a number of realizations of Cauchy and Dagum RFs and examine the wavefront evolutions according to (1). Thus, to evaluate how the RFs affect A_c we record the percentage of realizations that blow-up as a function of the initial condition. We also evaluate how the distance to blow-up changes as a function of initial condition. We do this by comparing x_∞'s coefficient of variation to the coefficient of variation of the random field. A total of 1024 realizations are generated for each α, β pair. If more than 128 realizations blow-up, then the statistics for x_∞ are calculated.

We further motivate this study as trying to answer the fundamental question of stochastic mechanics, when is it appropriate to assume a homogeneous system? We further describe this question below. For a deterministic system, the governing equation is of the form,

$$\mathcal{L}u = f \tag{4}$$

where $\mathcal{L}$ is some differential operator, u is the unknown solution, and f is a known source. In stochastic systems with a random medium, noise is introduced to the differential operator,

$$\mathcal{L}(\omega)u = f, \tag{5}$$

where ω is a realization of sample space Ω and indicates randomness. Ω is defined over a probability space $(\Omega, \mathcal{S}, P)$, where $\mathcal{S}$ is a σ-algebra, and P denotes a Gaussian measure. That is to say, $\mathcal{L}$ governs the response of a random medium $\mathcal{B}$. The random medium is sampled from the set of all possible realizations $B(\omega)$ parameterized by an event ω of the Ω space,

$$\mathcal{B} = \{B(\omega);\ \omega \in \Omega\}. \tag{6}$$

The goal is to find $\langle u\rangle$ and other statistics like higher order moments. The angle brackets denote the stochastic mean or expectation which is given by,

$$\langle u\rangle = \int_\Omega u \, \mathrm{d}P. \tag{7}$$

$\langle u\rangle$ is typically found from multiplying both sides of Eq. (5) by the inverse of $\mathcal{L}$ and applying the stochastic mean to each side, which is given as,

$$\langle u\rangle = \langle \mathcal{L}^{-1}\rangle f \text{ or } \langle \mathcal{L}^{-1}\rangle^{-1}\langle u\rangle = f, \tag{8}$$

where the -1 superscript denotes the inverse. However, explicitly solving Eq. $(8)_2$ is generally unfeasible. As a result, one typically resorts to replacing Eq. $(8)_2$ with,

$$\langle \mathcal{L}\rangle\langle u\rangle = f. \tag{9}$$

Basically, when is it suitable to replace Eq. $(8)_2$ with (9)? In simpler terms, when can we assume a homogeneous medium? We now want to address this issue for propagation of acceleration waves where the random medium, $\mathcal{B}$, is applied to the dissipation and non-linearity, μ and λ, respectively for two different cases. The first case is for two variables with the same noise of varying intensity and the second case is for four variables with the same noise of varying intensity.

Previous studies for acceleration waves in random media exist [10–13]. Ref. [10] looks at acceleration waves subject to white noise random fields and Ornstein–Uhlenbeck process and their effect on A_c. Other studies [11] and [12], investigate the influence of white noise on x_∞ as well as comparing the effect of correlations of μ and λ; see [14] for a review of results up to 2008. Ref. [13] looks at the random fields' effects for shocks in viscoelastic media.

In this article, we study the effects of randomness with fractal dimension and Hurst exponent on the critical amplitude and distance to blow up. In Section 2, we introduce the RFs used here and the differences due to the fractal dimension and Hurst exponent. In Section 3, we introduce randomness to our governing equation, Bernoulli equation, and we introduce the numerical methodology. Section 4, presents the results.