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ORIGINAL RESEARCH ARTICLE

Wave-induced bottom shear stress estimation in shallow water exemplified by using deep water wind statistics

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Summary The paper provides a simple and analytical method which can be used to give estimates of the wave-induced bottom shear stress for very rough beds and mud beds in shallow water based on wind statistics in deep water. This is exemplified by using long-term wind statistics from the northern North Sea, and by providing examples representing realistic field conditions. Based on, for example, global wind statistics, the present results can be used to make estimates of the bottom shear stress in shallow water.

1. Introduction

Simple and effective descriptions of transport mechanisms in operational estuarine, coastal and ocean circulation models are often required, in which the bottom shear stress represents an important component in finite water depths. In estuarine and coastal zones, at shallow and intermediate water depths, the water particle movements induced by surface waves have a strong effect in the entire water column from the surface to the bottom of the sea. The flow in this region is generally induced by surface waves and currents, where the bottom wave boundary layer is a thin flow region at the seabed dominated by friction arising from the bottom roughness. The wave boundary layer flow determines the bottom shear stress, which affects, e.g. the sediment transport and assessment of the stability of scour protections in the marine environment. The boundary layer flow regime is most commonly rough turbulent, although the flow regime over mud beds is mostly laminar and smooth turbulent depending on the bottom sediments and wave activity.

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The rough turbulent flow regime considered here corresponds to very rough beds. The results in this flow regime are relevant for assessing, e.g. the stability of scour protection in the coastal environment for relative large stone sizes compared to the near-bed random wave activity.

Laminar flow near mud beds, where clays and silt are referred to as mud, is of practical interest. The movement of mud within coastal and estuarine waters might have large economical and ecological impact in the development of new engineering works and maintenance of existing installations, e.g. related to necessary routine dredging required for ports' accessibility to shipping. The capability to predict the movement of the mud is also essential to understand the distribution of certain pollutants adsorbed to mud, as cohesive sediments are often contaminated. It appears that organic (polychlorinated biphenyl (PCBs), etc.) pollutants adhere easily to the clay particles and organic materials of the sediments. The results for laminar flow are relevant for assessing erosion and deposition of mud beneath random waves.

Further details on the background and complexity of the flow, as well as reviews of the problems are found in the textbooks of, e.g. Nielsen (1992), Fredsøe and Deigaard (1992), Soulsby (1997), Whitehouse et al. (2000), Winterwerp and van Kesteren (2004).

The purpose of this study is to demonstrate how wind statistics in deep water can be used to provide the wave-induced bottom shear stress in shallow water. Results are given for the bottom shear stress beneath random surface waves at beds with very large roughness and for laminar flow applied to mud beds, and are primarily based on the previous work by Myrhaug and Holmedal (2010) who provided the seabed shear stress spectrum for very rough beds and for laminar flow. Examples of results representing realistic field conditions are given.

2. Bottom shear stress beneath random waves in shallow water

2.1. Spectrum of bottom shear stress

Following Myrhaug and Holmedal (2010) (hereafter referred to as MH10) the bottom shear stress spectrum for laminar flow in shallow water ($kh \ll 1$) is obtained as (see Eq. (A12) in the Appendix)

$$S_{(\tau/\rho)(\tau/\rho)}(\omega,h) = \frac{\nu_f \omega^3}{(kh)^2}\,\frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \tag{1}$$

Here τ is the bottom shear stress, ρ is the fluid density, ω is the cyclic wave frequency, h is the water depth, ν_f is the kinematic viscosity of the fluid, k is the wave number determined from the dispersion relationship $\omega^2 = gk\tanh kh$ which in shallow water reduces to $\omega^2 = k^2 gh$, g is the acceleration due to gravity, and $S_{\zeta\zeta}(\omega)$ is the deep water wave spectrum.

The bottom shear stress spectrum for rough turbulent flow over a bed with very large roughness in shallow water is obtained as (see Eq. (A16) in the Appendix)

$$S_{(\tau/\rho)(\tau/\rho)}(\omega,h) = \frac{c^2 z_0^2 \omega^4}{4(kh)^2}\,\frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \tag{2}$$

Here z_0 is the average bottom roughness, and c is a constant with the two values 9 and 18 reflecting that c depends strongly on the geometry of the large roughness elements (see the Appendix for more details). The first term on the right hand side of Eqs. (1) and (2) represents the square of the magnitude of the transfer function between the bottom shear stress τ/ρ and the free surface elevation ζ; the second term represents the depth correction factor in shallow water, i.e. a correction factor which is used to transform the deep water wave spectrum $S_{\zeta\zeta}(\omega)$ to shallow water (see the Appendix for more details).

By substituting $k^2 = \omega^2/gh$, Eqs. (1) and (2) are rearranged, respectively, to

$$S_{(\tau/\rho)(\tau/\rho)}(\omega,h) = \frac{1}{2}\nu_f \omega^3 S_{\zeta\zeta}(\omega); \quad \text{laminar}, \tag{3}$$

$$S_{(\tau/\rho)(\tau/\rho)}(\omega,h) = \frac{1}{8}(cz_0)^2 \omega^4 S_{\zeta\zeta}(\omega); \quad \text{rough}. \tag{4}$$

Thus the shear stress spectra in shallow water are given in terms of the deep water wave spectrum, and it should be noted that the dependence on the water depth disappears. Overall this is a consequence of transforming the waves from deep to shallow water and using the bed shear stress for laminar flow (Eqs. (A3) and (A8)) and for very rough beds (Eqs. (A3) and (A13)).

2.2. Laminar flow

The zeroth spectral moment of the bottom shear stress spectrum for laminar flow is obtained from Eq. (3) as

$$m_{0\tau/\rho} = \int_0^\infty S_{(\tau/\rho)(\tau/\rho)}(\omega,h)\,d\omega = \frac{\nu_f}{2} m_3, \tag{5}$$

where m_3 is the third wave spectral moment in deep water, i.e. the nth spectral moment of the deep water wave spectrum is defined as

$$m_n = \int_0^\infty \omega^n S_{\zeta\zeta}(\omega)\,d\omega; \quad n = 0,1,2,3,4,\ldots \tag{6}$$

Thus, from Eq. (5) the significant value of the bottom shear stress height is obtained as

$$H_{s\tau/\rho} = 4\sqrt{m_{0\tau/\rho}} = 2\sqrt{2\nu_f m_3}. \tag{7}$$

2.3. Very rough beds

The zeroth spectral moment of the bottom shear stress spectrum for very rough beds is obtained from Eq. (4) as

$$m_{0\tau/\rho} = \frac{1}{8}(cz_0)^2 m_4, \tag{8}$$

where m_4 is the fourth spectral moment of the wave spectrum in deep water defined in Eq. (6). The most common model wave spectra are proportional to ω^{-5} for large ω, and thus m_4 does not exist. However, m_4 can be expressed in terms of the spectral moments m_0, m_1 and m_2, and the