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Quantum field theory and coalgebraic logic in theoretical computer science

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ABSTRACT

We suggest that in the framework of the Category Theory it is possible to demonstrate the mathematical and logical *dual equivalence* between the category of the q-deformed Hopf Coalgebras and the category of the q-deformed Hopf Algebras in quantum field theory (QFT), interpreted as a thermal field theory. Each pair algebra-coalgebra characterizes a QFT system and its mirroring thermal bath, respectively, so to model dissipative quantum systems in far-from-equilibrium conditions, with an evident significance also for biological sciences. Our study is in fact inspired by applications to neuroscience where the brain memory capacity, for instance, has been modeled by using the QFT unitarily inequivalent representations. The q-deformed Hopf Coalgebras and the q-deformed Hopf Algebras constitute two dual categories because characterized by the same functor T, related with the Bogoliubov transform, and by its contravariant application T^{op}, respectively. The q-deformation parameter is related to the Bogoliubov angle, and it is effectively a thermal parameter. Therefore, the different values of q identify univocally, and label the vacua appearing in the foliation process of the quantum vacuum. This means that, in the framework of Universal Coalgebra, as general theory of dynamic and computing systems ("labelled state-transition systems"), the so labelled infinitely many quantum vacua can be interpreted as the Final Coalgebra of an "Infinite State Black-Box Machine". All this opens the way to the possibility of designing a new class of universal quantum computing architectures based on this coalgebraic QFT formulation, as its ability of naturally generating a Fibonacci progression demonstrates.

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1. Introduction

Category Theory (CT) endows modern logic and mathematics with a universal language in many senses "wider" than set theory. By CT indeed, it is possible to demonstrate structural similarities among theories that it is impossible to discover otherwise. In this framework, there exists a growing convergent interest in literature toward the *topological interpretation* of quantum field theory (QFT), both in theoretical physics and in theoretical computer science (for a comprehensive discussion on the role of CT providing links among quantum physics, topology, and quantum computation see, e.g., Rasetti and Marletto, 2014).

In this paper we present a further exemplification of the power of CT on this regard and we discuss a possible approach to topological quantum computing, in the framework of QFT interpreted as a thermal field theory, according to a development of the original Umezawa's Thermal Field Dynamics (TFD) (Takahashi and Umezawa, 1975; Umezawa et al., 1982; Umezawa, 1993). This development is based on the systematic use of the "q-deformed Hopf coalgebras" for modeling *open* quantum systems, where q is a thermal parameter related to the Bogoliubov transformation. Here it has to be observed that from a purely co-algebraic category setting standpoint, primitive co-product of algebras would be sufficient in a thermal field theory framework. Where the q-deformation becomes unavoidable is in the quantization procedure which requires canonicity, namely to consider the global (*closed*) system-environment setting in order to allow the proper definition of a complete set of conjugate variables necessary for the canonical quantization procedure (Celeghini et al., 1992, 1998). It is due to such a well definite process of 'closing' the system by inclusion in the formalism of its environment, that global topological aspects may enter, thus accounting for long-range correlations dynamically generated in the system-environment interaction. Under a general formal perspective, the interplay and the implications of the global *vs* local topology is discussed in Rasetti and Merelli (2017), where topological field theory powerfulness is exploited to face the mining data problem, which is of crucially growing relevance in many-body physics, in biology and neuroscience.

The coalgebras thus introduced are characterized by *non-commutative co-products*, since they represent states of the system and states of its thermal bath, and these cannot be treated on the same basis and cannot be exchanged one with the other. We thus introduce the *doubling of the degrees of freedom* (DDF), based on the doubling of the states of the Hilbert space, including the system states and the thermal bath states (Blasone et al., 2011). This opens the way to a topological QFT modeling of quantum dynamics for systems *far-from-equilibrium,* and not only in a *near-to-equilibrium* condition.

Our approach to a topological QFT is, therefore, a development of TFD as far as is based on the mechanism of spontaneous breakdown of symmetry (SSB) with the related Goldstone theorem, the Goldstone boson condensates (Goldstone et al., 1962) and the Nambu-Goldstone (NG) *long-range correlation* modes or NG quanta, familiar in condensed matter physics and elementary particle physics.

These NG correlations allow the existence of infinitely many *unitarily inequivalent representations* of the canonical commutation relations (CCR's) in QFT (unitarily inequivalent Hilbert spaces of the system states) (Blasone et al., 2011). The DDF allows then to use the *minimum free energy function and measure* as an intrinsic dynamical tool of choice among states, so to grant a dynamic determination of the orthonormal basis of the Hilbert space. It introduces at the same time the notion of the quantum vacuum (QV) or ground state *foliation*, as a robust principle of "construction" and of "memory" used by nature for generating ever more complex systems.

It is therefore not casual that during the last ten years the QV-foliation in QFT has been successfully applied to solve dynamically the capacity problem of the *long-term memories* – namely, the "deep beliefs" in the computer science jargon – in the living brain, interpreted as a "dissipative brain", i.e.,"entangled" with its environment (thermal bath) via the DDF formalism (Vitiello, 1995, 2001, 2015; Freeman and Vitiello, 2006, 2008; Capolupo et al., 2013; Basti, 2013).

These neurophysiological studies provide, on the one hand, the justification for our analysis in terms of QFT algebraic structures addressed to the formal description of some aspects of the brain functional activity and of far-from-the-equilibrium systems as biological systems are. On the other hand, they also suggest that the DDF based on a coalgebraic modeling of open systems in QFT could be, in computer science, an effective possible solution of the problem, so-called of "deep learning", arising with "big-data" modeling and, overall, in dealing with an effective computational management of (infinite) "streams" (think, for instance, at the internet streams) characterized by a continuous change of the "hidden" higher-order correlations among data, which are absolutely unpredictable by the classical statistical tools. In this case, the DDF formalism could endow a topological quantum computing system with a dynamic (= automatic) rearrangement of the dimensions (degrees of freedom) of the representation space of the system, so "to lock it dynamically" on the ever changing degrees of freedom of the stream (interpreted as the system thermal bath), by using the minimum free energy function as a truth evaluation function (Basti, 2017). Here, again it appears the central role of the interplay global *vs* local character of q-algebraic category (Rasetti and Merelli, 2017). Although, as observed above, the global topological aspects play a crucial role, the local aspects show their relevance in dealing with infinite data streams. This reminds us of the interplay, in a different perspective and context, between global and local invariance in gauge theories, where SSB can only occur for the global gauge symmetry, with consequent formation of long-range correlation modes (NG boson modes), while gauge fields involved in local gauge transformations manifest themselves as massive excitations (the Anderson-Higgs-Kibble mechanism).

Effectively, one of the pillars of the topological quantum computing, i.e., the fundamental *Stone representation theorem for Boolean algebras* (Stone, 1936), is strictly related with the topological interpretation of QFT. Stone, indeed, arrived at this theorem after a deep reflection on Hilbert spectra that leaded him to demonstrate five years before in 1931, with J. von Neumann, a fundamental theorem at the basis of the formalism for QM we discuss below (von Neumann, 1955). The topologies of the Stone spaces are effectively the same topologies of the C^*-algebras, and

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