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Robustness Analysis of the Scale-Free Networks

Jianhua Zhang*, Bo Song, Zhaojun Zhang, Mingwei Zhao

School of electrical engineering and automation, Jiangsu Normal University, Xuzhou, China

Abstract

In this paper, we investigate the connectivity of the scale-free networks and introduce degree variance which describes the numerical distance between degree and the average degree of vertices of the scale-free networks, and the degree variance is looked as a measure on robustness of the scale free networks subjected to failures. According to degree variance, we investigate the robustness of the scale free networks subject to random failures and intentional attacks, and discuss the vertices' ability to defense the cascading failures. Furthermore, when the average degree is given, a scheme of nonlinear integer program is designed to defense the random and intentional attacks according to degree variance. From the scheme, we can obtain the optimal robustness to cascading failures, and the scheme can be adopted to improve robustness of scale-free networks.

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Keywords: Scale-free networks; Robustness; Degree variance; Nonlinear integer program

1. Introduction

Complex networks have attracted more and more attention in the past several years [1, 2, 3, 4, 5], the most important reason of this phenomenon is due to its wide existence and application in reality, such as the internet, economic system, electrical grid, biological system, transportation network and so on, they are all essential to the modern society. Because the security of the infrastructural networks influences many aspects

* Corresponding author. Tel.: +86 051683500273; fax: +86 051683500273.
E-mail address: zhangjianhua1980@126.com.

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in our modern lives, so many researchers have taken part in this research field and have obtained many valuable results [6, 7, 8, 9]. Since the discovery of the small-world networks, their behaviours [2] have been investigated from many aspects, and researchers also construct many actual complex models from reality. There are many scale-free networks in reality, and the scale-free property [3] have attracted continuous attention [10, 11, 12] from all over the world, and many networks have a scale-free connectivity distribution [13, 14] which means that degree distribution of the scale-free networks follows a power-law distribution [4, 5, 6]. So many characteristics of the scale-free networks have been discovered according to density function of degree. From Motter *et al.* [15], we know that the node i of the complex network has a load capacity to cascading failures, as follows,

$$C_i = (1+\lambda)L_i, \quad i = 1, 2, \cdots, N$$

where $\lambda > 0$ is the tolerance parameter, L_i is the initial load of the node i, and N is the total number of nodes. Although this assumption cannot completely reflect the fact that the capacity will be variable as the change of time in several networks, it can tell us many universal laws, such as, how to design the scheme to improve the robustness of networks subjected to random or intentional attacks. It can also give us inspirations to construct a better model to measure the robustness of networks, and describe the resistances of nodes to attacks.

Recently, there were many novel concepts and schemes in many fields, such as information science and engineering, statistical and nonlinear physics, systematic biology, and mathematics and social science are applied to investigate approaches to settle problems, especially in complex network [7, 8, 9]. The most important point of the robustness of complex networks is that the flow of information between topological vertices and other physical quantities, and this flow is critical to network's security, such as GPS, national defence, military security, eco-system security and supplying chain security. The security of complex networks will affect our behaviours from various aspects, so we must design a better scheme to measure the robustness of networks and keep the information exchange unimpeded in systems. The works by Albert *et al.* [3-4] demonstrated that scale-free networks possess the robust-yet-fragile property, which means that it is robust against random failures but fragile to intentional attacks. So we will design a model which can improve the resistances of networks subject to random failures or intentional attacks. Failures to nodes are serious threats to complex networks, it can change the balance of flow and lead to redistributing loads all over the network, and sometimes it can also cut down the information transmissions in the whole network [15]. Furthermore, the slight failure can cause cascading failures and the networks will be breakdown sometimes.

The paper is organized as follows. In Section 2, we discuss the connectivity which is looked as a measure about robustness of the scale-free networks. Degree variance is introduced in Section 3 and the optimal robustness of the scale-free networks is investigated in Section 4. Finally, conclusion is given in Section 5.

2. Connectivity of scale-free networks

In this section, we investigate the connectivity of the scale-free networks. We know there are many measures about the robustness of complex networks, and different measures have different characteristics in estimating the robustness of complex networks. In this paper, we introduce another measurement of the scale-free networks, which is called connectivity coefficient, as follows,

$$S = n(l) / N \tag{1}$$

where $n(l)$ is the total number of edges of topology. From the definition about the connectivity coefficient, we know that it can be regarded as a measure of the scale-free networks, and it can uncover the global connectivity of networks. From the properties of complex networks and our scheme, we know that the larger the connectivity coefficient is, the better the robustness is, so it is a good measure about the robustness. According to [16, 17], the connectivity coefficient of the scale-free networks can be obtained by,