Biomedical Signal Processing and Control 25 (2016) 196–200

Contents lists available at ScienceDirect

Biomedical Signal Processing and Control

journal homepage: www.elsevier.com/locate/bspc

Short Communication

A regularised EEG informed Kalman filtering algorithm

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ARTICLE INFO

Article history:
Received 2 June 2015
Received in revised form 8 October 2015
Accepted 10 November 2015
Available online 24 December 2015

Keywords:
Electroencephalogram
Informed Kalman
Voluntary movement prediction
State detection

ABSTRACT

The conventional Kalman filter assumes a constant process noise covariance according to the system's dynamics. However, in practice, the dynamics might alter and the initial model for the process noise may not be adequate to adapt to abrupt dynamics of the system. In this paper, we provide a novel informed Kalman filter (IKF) which is informed by an extrinsic data channel carrying information about the system's future state. Thus, each state can be represented with a corresponding process noise covariance, i.e. the Kalman gain is automatically adjusted according to the detected state. As a real-world application, we demonstrate for the first time how the analysis of electroencephalogram (EEG) can be used to predict the voluntary body movement and inform the tracking Kalman algorithm about a possible state transition. Furthermore, we provide a rigorous analysis to establish a relationship between the Kalman performance and the detection accuracy. Simulations on both synthetic and real-world data support our analysis.

1. Introduction

Kalman filtering (KF) is a popular state estimation technique which has found a wide range of applications in science and technology. It provides optimal error correction for noisy and inaccurately modelled random processes through a recursive algorithm which accumulates information regarding the process characteristics. Recently, the Kalman algorithm has been formulated in the quaternion domain representation to track in three-dimensional spaces [1–3].

The Kalman algorithm requires prior knowledge of the system such as the system model, its initial conditions and the noise characteristics to provide a robust performance. The conventional Kalman filter algorithm considers processes where the noise characteristics or the system dynamics remain stationary [2,4]. However, those characteristics may change in their structure and behaviour, such as random system failures, environmental disturbances and abrupt variation of the operating point [5]. Thus, the conventional Kalman filter may not be able to capture those changes resulting in suboptimal performance. For instance, consider a motion tracking system where there are periods in which the motion dynamics are of either low or high variance. In this case, a stationary process noise model is not optimal [5,6].

The performance of the Kalman filter depends on the Kalman gain, a parameter which provides a tradeoff between the actual observations and the model predictions. The Kalman gain is computed based on the noise characteristics and determines the performance of the KF. In a motion tracking system, a Kalman gain can be tuned to achieve noise reduction behaviour by assuming a low value and responsiveness by assuming a large value. Since the Kalman gain is directly affected by the process noise covariance, the correct estimation of this matrix can significantly enhance the robustness and reliability of the KF.

These problems have been addressed in [6], where a state-based gain adaptation algorithm was developed in which the Kalman gain depended on the observed measurements. However, in practice, online identification of the transition point is very challenging, especially when the data contain high level of noise. In this work, we propose an informed Kalman filter (IKF) algorithm where the Kalman gain is automatically updated based on an extrinsic data channel which provides information about the state of the system. Thus, optimal behaviour can be established for systems exhibiting non-stationary changes in the system model, its dynamics and in the noise behaviour. The external data channel operates as a predictor of the future evolution of the system's parameters and its prediction accuracy is critical for the IKF efficacy.

We demonstrate a practical application of the IKF algorithm by considering motion tracking of human arm movements and the extrinsic data channel corresponds to concurrent electroencephalography (EEG) measurements. An early indication of volitional movement is the pre-motor or readiness potential (RP), which appears about 0.5–1.5 s prior to the initiation of voluntary movement [7–10].

The RP is known as part of the slow-wave motion related cortical potential (MRCP) which is related to the movement planning and

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http://dx.doi.org/10.1016/j.bspc.2015.11.005
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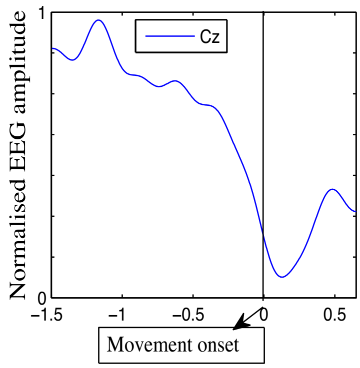

Fig. 1. The obtained readiness potential for a healthy subject. The RP wave is in agreement with literature [7].

execution. The RP wave is represented as a slow decrease in EEG amplitude starting about 1.5 s prior to the movement onset [7,8] where the decreasing rate reaches a steep slope about 0.4 s before the movement, see Fig. 1.

Thus, we aim to detect the online RP prior to motion execution and impart it to the proposed IKF as the system change predictor. Note that online detection of the RP wave from single-trials has always been a challenge due to the poor signal to noise ratio (SNR) of the EEG [10]. Furthermore, the onset and appearance of the RP wave can differ among participants and movement conditions. This is due to several factors such as level of intention, preparatory state, speed and precision of movement, pace of movement repetition, complexity of movement, and pathological lesions of various brain structures [8]. In this work, we provided a uniform preparation state for subjects and they were asked to perform a similar arm movement. The RP wave was detected using an individual template-matching algorithm.

The design of the IKF in this work employs a quaternion representation for the motion tracking data but the same principles apply for real or complex data.

2. Methodology

Recently, Kalman filtering has been formulated within the quaternion domain to address 3-D altitude estimation problems [1]. Similar to the conventional KF, the quaternion KF is a recursive algorithm that consists of two major steps, prediction and update. The KF behaviour is often discussed in terms of $\mathbf{K}_0$ or Kalman gain, a correction factor for state estimation, which is affected by both process noise ($\mathbf{Q}$) and measurement noise ($\mathbf{R}$) matrices. Thus, optimising the system to have reasonable values for covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ is required for optimal performance.

2.1. EEG informed Kalman gain

In this section, we introduce an informed Kalman algorithm in which the Kalman gain is adjusted according to the state of the system. Consider the system $\mathbf{S}_r$ which represents a random mixture of low and high variance dynamics represented by two process noise covariances $\mathbf{Q}_1$ and $\mathbf{Q}_2$ respectively. For this system, $\mathbf{Q}_2$ is preferred when noise reduction is the main objective, while $\mathbf{Q}_1$ is applied for higher responsiveness of the KF to abrupt changes. Thus, we propose the IKF in which the combined gain leverages both of these performance advantages according to the state of the data. The gain is derived from $\mathbf{K}_1$ and $\mathbf{K}_2$, thereby taking into account the noise statistics $\mathbf{Q}_1$ and $\mathbf{Q}_2$, and a regularisation parameter α which reflects the state variance. In other words, α is defined such that $\mathbf{K}_2$ is the principal gain for low variance movements to denoise the data, while $\mathbf{K}_1$ is the primary gain for high variances, such

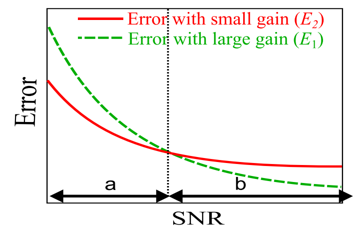

Fig. 2. Illustration of Kalman error vs. SNR for two different noise covariance matrices, where $\mathbf{Q}_1$ (dotted line) > $\mathbf{Q}_2$ (solid line).

as occurrence of sudden changes, to compensate for the delayed response. The derivation of the novel Kalman gain in terms of the system characteristics is included in Appendix A.

In practice, online identification of the transition point is challenging, especially when the data contain high level of noise. Therefore, we assume that this instant is highly correlated with a known distinct feature in an extramural channel, such as RP wave in EEG before the movement execution. Thus, rather than the actual data, we exploit an extrinsic EEG channel ($\mathbf{x}$) to inform the KF for state recognition and gain adaptation. The IKF[1] is summarised in Algorithm 1.

Algorithm 1. Informed Kalman filter
Initialisation of the Kalman variables
Kalman state prediction
$\tilde{\mathbf{q}}_p(t+1) = \mathbf{F}\tilde{\mathbf{q}}_u(t)$
$\mathbf{P}_{p_i}(t+1) = \mathbf{F}\mathbf{P}_{u_i}(t)\mathbf{F}^T + \mathbf{Q}_i \quad i \in \{1, 2\}$
$\mathbf{K} = \alpha\mathbf{K}_1 + (1-\alpha)\mathbf{K}_2 \quad \text{where} \quad \mathbf{K}_i = \mathbf{P}_{p_i}(t+1)\mathbf{H}^T(\mathbf{H}\mathbf{P}_{p_i}(t+1)\mathbf{H}^T + \mathbf{R})^{-1} \quad i \in \{1, 2\}$
Kalman update
$\tilde{\mathbf{q}}_u(t+1) = \tilde{\mathbf{q}}_p(t+1) + \mathbf{K}(\mathbf{q}_{os} - \mathbf{H}\tilde{\mathbf{q}}_p(t+1))$
$\mathbf{P}_{u_i}(t+1) = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_{p_i}(t+1) \quad i \in \{1, 2\}$

Note that $0 \le \alpha \le 1$ is the regularisation factor. On one hand, $\alpha = 1$ provides maximum convergence and highest responsiveness, on the other hand $\alpha = 0$ is optimal for the low-variance state; and therefore $\alpha > 0$ models the transient state. In this work, the value of α is affected by detection of the RP which is achieved via a template matching obtained from the training EEG data.

3. IKF performance analysis

To analyse the performance of proposed IKF against noise, assume that for the system $\mathbf{S}_r$, both $\mathbf{Q}$ covariance matrices are diagonal where $\mathbf{Q}_1 > \mathbf{Q}_2$. Theoretically, for small SNR, e.g. region (a) in Fig. 2, the noise reduction is more crucial than responsiveness and small Kalman gain leads to a lower error or E_2. On the other hand, for high SNR, region (b) in Fig. 2, noise is less dominant and quick adaptation of the Kalman algorithm is desired. Thus, the error of high Kalman gain or E_1 is smaller.

Using the proposed IKF, the objective is to leverage both of these performance advantages regardless of SNR, such that the performance shifts towards the optimal case. However in practice, the exterior predictor channel is noisy and this affects the state detection and consequently the IKF behaviour. Therefore, the IKF performance should be also examined in terms of detection accuracy of events in the predictor channel (EEG). For statistical analysis, a most common index of the accuracy is associated with a receiver operating characteristic (ROC) curve. This curve is acquired by plotting the true positive rate (TPR) versus false positive rate (FPR) for various detection thresholds [7]. In general, the final threshold is

[1] Throughout this paper, $\mathbf{H}$ represents the observation model to map the state space into the observed space, and $\mathbf{F}$ is a matrix of state model to illustrate the system's dynamic. The $\tilde{\mathbf{q}}_p$ and $\tilde{\mathbf{q}}_u$ are the priori (predict) and posteriori (update) states respectively and $\mathbf{q}_{os}$ is the observed state.