Mechanical Systems and Signal Processing 66-67 (2016) 78–88

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

An inner–outer nonlinear programming approach for constrained quadratic matrix model updating ☆

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ARTICLE INFO

Article history:
Received 1 September 2014
Received in revised form
18 March 2015
Accepted 1 May 2015
Available online 1 June 2015

Keywords:
Quadratic matrix model updating
Quadratic Rayleigh quotient
Nonlinear programming
Algorithms

ABSTRACT

The Quadratic Finite Element Model Updating Problem (QFEMUP) concerns with updating a symmetric second-order finite element model so that it remains symmetric and the updated model reproduces a given set of desired eigenvalues and eigenvectors by replacing the corresponding ones from the original model. Taking advantage of the special structure of the constraint set, it is first shown that the QFEMUP can be formulated as a suitable constrained nonlinear programming problem. Using this formulation, a method based on successive optimizations is then proposed and analyzed. To avoid that spurious modes (eigenvectors) appear in the frequency range of interest (eigenvalues) after the model has been updated, additional constraints based on a quadratic Rayleigh quotient are dynamically included in the constraint set. A distinct practical feature of the proposed method is that it can be implemented by computing only a few eigenvalues and eigenvectors of the associated quadratic matrix pencil.

1. Introduction

The Quadratic Finite Element Model Updating Problem (QFEMUP) concerns with updating a finite-element generated model of a vibrating structure of the form

$$M\ddot{x}(t)+D\dot{x}(t)+Kx(t)=0, \tag{1}$$

where M, D, and K are real $n\times n$ matrices known as mass, damping, and stiffness, respectively; and $\dot{x}(t)$ and $\ddot{x}(t)$ denote the first and second derivatives of the time-dependent vector $x(t)$. The eigenvalues of the associated quadratic pencil

$$Q(\lambda)=\lambda^2 M+\lambda D+K \tag{2}$$

are related to natural frequencies and the eigenvectors are the mode shapes of the vibrating system (1) (see, e.g., [18,19,29]). The quadratic pencil (2) has $2n$ eigenvalues and $2n$ eigenvectors. The dynamics of the system are modeled by these

☆ This work was supported by PRONEX-CNPq/FAPERJ (E-26/111.449/2010-APQ1), FAPESP (Grants 2010/10133-0, 2013/03447-6, 2013/05475-7, and 2013/07375-0), and CNPq (PVE 71/2013, Proj. 400926/2013-0 and 476792/2013-4).
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http://dx.doi.org/10.1016/j.ymssp.2015.05.002
0888-3270/

eigenvalues and eigenvectors. For example, it is well-known that the stability of a vibrating system is determined by the nature of a few dominating natural frequencies. It is also well-known that sometimes the vibrating structures experience dangerous vibrations, called *resonance*, when a natural frequency becomes close or equal to a frequency of an external force, such as earthquake, gusty wind, weights of the human bodies, among others. Failures of many structures like buildings, bridges, airplane wings, and turbines have been attributed to resonance.

Eq. (1) is usually obtained by discretization of a distributed parameter system with finite element techniques, and therefore, known as the finite element model. The matrices M, D, and K are often very large and sparse but have some structure, such as M is symmetric and positive definite ($M = M^T > 0$) and often diagonal, and D and K are symmetric ($D = D^T$ and $K = K^T$).

The QFEMUP consists in updating the quadratic pencil $Q(\lambda)$ to another quadratic pencil

$$\tilde{Q}(\lambda) = \lambda^2 M + \lambda \tilde{D} + \tilde{K} \tag{3}$$

in such a way that a small number $1 \le p < 2n$ of given measured eigenvalues and eigenvectors from a real-life structure or an experimental structure are reproduced by the updated pencil. Besides the basic requirements of preserving the symmetry and sparsity pattern of the new matrices $\tilde{D}$ and $\tilde{K}$ and reproducing the p measured eigenvalues and eigenvectors, there are certain other engineering issues that must be taken into account while solving the problem in practice. For instance, it is important that the new matrices $\tilde{D}$ and $\tilde{K}$ are as close as possible to the original ones D and K, respectively, which imposes an optimization approach. It is also very important that no spurious modes appear in the frequency range of interest after the model has been updated; see [25]. The so-called no spill-over constraint which, assuming that the p eigenpairs to be replaced are known, forces the additional $2n-p$ eigenvalues and corresponding eigenvectors to remain unchanged, clearly guarantees that no spurious modes will appear in the frequency range of interest. Several numerical schemes have been recently proposed to accomplish all the mentioned requirements, including the no spill-over constraint, for several different scenarios; see, e.g., [6,12–17,21,22,23,24,29-31,35] and references in there. In most cases, the no spill-over constraint is accomplished by using some clever linear algebra theoretical results that involve the solution of several large-scale matrix equations (Lyapunov, Sylvester, and block linear systems); see, e.g., [13,14,31,35].

In several important applications, instead of forcing the no spill-over constraint, what is important from a practical point of view is to guarantee that spurious modes are not introduced into the frequency range of interests (see [25]). For this scenario, we present in this paper a new optimization approach that not only maintains the symmetry, the sparsity structure, and the nearness of the matrices $\tilde{D}$ and $\tilde{K}$, while reproducing the p measured eigenvalues and eigenvectors, but also pays special attention to the fundamental engineering requirement of making sure that no spurious modes appear in the frequency range of interest. In this work, we accomplish all these requirements without forcing the no spill-over constraint. For that, our new scheme combines an optimization procedure with the dynamical inclusion of additional constraints in an inner–outer iterative scheme. The additional constraints are based on a suitable recent extension of the Rayleigh quotient for quadratic eigenvalue problems [27,36]. A key practical feature of the proposed scheme is that it can be implemented by computing only a few additional eigenvalues and eigenvectors of the associated quadratic matrix pencil.

Variations of finite element model updating problems, with different levels of difficulty, have been solved in the past using iterative numerical optimization techniques of several types and for different objective functions; see, e.g., [1,6–8,12,20,34]. In particular, the ones that guarantee the no spill-over constraint need to solve several large-scale matrix equations for each function and gradient evaluation, which require a computational cost of $O(pn^3)$ floating point operations (flops) per iteration; see, e.g., [6,12].

The rest of the paper is organized as follows. In Section 2, we formulate the QFEMUP as a constrained optimization problem and describe the variables and the constrains, including the way of forcing the matrices' sparsity structure and symmetry. In Section 3, we describe the suitable use of the Rayleigh quotient for quadratic eigenvalue problems for building the constraints or cuts, to avoid when necessary the presence of spurious modes in the frequency range of interest. We also describe in detail the inner–outer iterative scheme, and discuss its theoretical properties. In Section 4, we show the performance of our scheme on some illustrative examples. Concluding remarks are presented in Section 5.

2. Mathematical programming formulation

Consider the quadratic pencil $Q(\lambda)$ given by (2), where $M, D, K \in \mathbb{R}^{n \times n}$ are given matrices such that M is symmetric positive definite and D and K are symmetric. Let $1 \le p < 2n$, $\lambda_i \in \mathbb{C}$, and $x_i \in \mathbb{C}^n$ be such that (λ_i, x_i) for $i = 1, \ldots, p$ are the desired eigenpairs. The goal is to find matrices $\tilde{D}, \tilde{K} \in \mathbb{R}^{n \times n}$ such that (λ_i, x_i) are eigenpairs of the updated quadratic eigenpencil $\tilde{Q}(\lambda)$ given by (3), i.e.,

$$(\lambda_i^2 M + \lambda_i \tilde{D} + \tilde{K}) x_i = 0, \quad i = 1, \ldots, p. \tag{4}$$

Matrices $\tilde{D} = (\tilde{d}_{ij})$ and $\tilde{K} = (\tilde{k}_{ij})$ must be symmetric and must preserve the sparsity pattern of $D = (d_{ij})$ and $K = (k_{ij})$, respectively. In addition, $\tilde{D}$ and $\tilde{K}$ must be as close as possible to D and K, respectively.

Let I_D and I_K be the sets of indexes of non-zero elements in the upper triangle of the given matrices D and K, respectively, i.e.,

$$I_D = \{(i,j) | 1 \le i \le j \le n \text{ such that } d_{ij} \ne 0\} \tag{5}$$