Mechanical Systems and Signal Processing 54-55 (2015) 108–123

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

An alternative method to the identification of the modal damping factor based on the dissipated energy

D. Montalvão [a,b,*], J.M.M. Silva [b]

[a] School of Engineering and Technology, University of Hertfordshire, College Lane Campus, Hatfield AL10 9AB, United Kingdom
[b] IDMEC/IST – Institute of Mechanical Engineering, Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

ARTICLE INFO

Article history:
Received 2 November 2013
Received in revised form
28 August 2014
Accepted 31 August 2014
Available online 18 September 2014

Keywords:
Experimental modal analysis (EMA)
Modal identification
Method of the inverse
Dissipated energy

ABSTRACT

The identification of the modal parameters from frequency response functions is a subject that is not new. However, the starting point often comes from the equations that govern the dynamic motion. In this paper, a novel approach is shown, resulting from an analysis that starts on the dissipated energy per cycle of vibration. Numerical and experimental examples were used in order to assess the effectiveness of the proposed method. It was shown that, for lightly damped systems with conveniently spaced modes, it produced quite accurate results when compared to those obtained from the method of the inverse. The technique also proved to be simple enough to be used for quick estimates of the modal damping factors. Finally, this paper is a contribution to modal analysis and identification methods, as the developed technique has never been proposed before.

1. Introduction

Modal identification seeks to obtain the global and local characteristics of vibrating structures using experimental data. This technique may be used either to obtain the global characteristics (natural frequencies and damping), to directly derive a mathematical model of the structure or to improve an existing finite element model through what is frequently called *updating*. The interest of modal identification procedures is acknowledged by the scientific community and many authors have addressed this problem, mainly since the early seventies of the past century [1]. The proposed modal identification procedures cover different levels of sophistication and, in almost all cases, need the use of special software that may not be easy to obtain.

In the past few years, attention has been more focused on Operational Modal Analysis (OMA) rather than in the more traditional Experimental Modal Analysis (EMA). Examples of later developments in OMA identification methods can be found, for instance, in [2–5]. In terms of EMA, later publications are more concerned with Engineering applications, as can be seen, for instance, in [6,7]. OMA deals with operational deflection shapes and many often make use of output-only measurements, this meaning that excitation loads are unknown. EMA makes use of both input forces and output responses in order to determine modal parameters and mode shapes. Numerous modal identification algorithms have been developed

* Corresponding author at: School of Engineering and Technology, University of Hertfordshire, College Lane Campus, Hatfield AL10 9AB, United Kingdom. Tel.: +44 1707 285387.
E-mail address: d.montalvao@herts.ac.uk (D. Montalvão).

http://dx.doi.org/10.1016/j.ymssp.2014.08.025
0888-3270/

in the past thirty years [8]. However, even if in the past recent years not many advances have been seen in terms of EMA modal identification methods, there are a few interesting results that can still be derived.

If the sole objective is the determination of the global modal characteristics, it is possible to use simple approaches producing quick estimates of the desired information. This issue is addressed in this paper where a new simple method is proposed, based on the dissipated energy per cycle of vibration. The proposed methodology is a robust estimator, provided the systems under analysis are not heavily damped and the modes are sufficiently separated so that their mutual interference may be assumed as negligible.

This paper presents the proposed new methodology and applies it to both numerical and experimental examples, showing that it yields reasonably accurate results.

2. Theoretical development

2.1. Definitions

The concept of a complex stiffness in vibration problems with viscous or structural (hysteretic) damping is something that has been known for decades. Most often the complex stiffness is defined as the sum of the stiffness itself (k, real part) and the damping coefficient (d, imaginary part):

$$k^* = k + id \tag{1}$$

To find the real and imaginary parts of the complex stiffness, it is easier if the more conventional viscous damping model is firstly introduced. The well-known second order differential equation of motion - for a single degree-of-freedom system (SDOF) – is given by:

$$m\ddot{x} + c\dot{x} + kx = Fe^{i\omega t} \tag{2}$$

where m is the mass, c is the viscous damping coefficient, k is the stiffness, F is the amplitude of the oscillatory force and t is the time variable. When excited by an harmonic force with frequency ω, it can easily be proven (and most fundamental texts on vibration theory show it, for instance [1,9]) that for each vibration cycle the system dissipates – through its viscous damper – a quantity of energy directly proportional to the damping coefficient, the excitation frequency and the square of the response amplitude X:

$$W_{diss} = \int_0^T f\dot{x}dt = \pi c\omega X^2 \tag{3}$$

where $T = 2\pi/\omega$ is the time period of oscillation. However, experimental evidence from tests performed on a large variety of materials show that the damping due to internal friction (material hysteresis) is nearly independent of the forcing frequency but still proportional to the square of the response amplitude [10], i.e.

$$W_{diss} \propto CX^2 \tag{4}$$

where C is a constant. Therefore, from Eqs. (3) and (4) the equivalent damping coefficient is:

$$c = \frac{C}{\pi\omega} = \frac{d}{\omega} \tag{5}$$

with $d = C/\omega$. In such conditions, Eq. (2) can be re-written as:

$$m\ddot{x} + \frac{d}{\omega}\dot{x} + kx = Fe^{i\omega t} \tag{6}$$

As $\dot{x} = i\omega x$ for a harmonic vibration, the previous equation may be re-written as:

$$m\ddot{x} + k(1 + i\eta)x = Fe^{i\omega t} \tag{7}$$

where

$$\eta = d/k \tag{8}$$

is known as the hysteretic damping ratio or damping loss factor. The quantity:

$$k^* = k(1 + i\eta) \tag{9}$$

is the same complex stiffness as initially described in Eq. (1).

The latter formulation (7) leads to the conclusion that the dissipated energy per cycle of vibration is independent of the forcing frequency.