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Optimal linear estimators for multi-sensor stochastic uncertain systems with packet losses of both sides

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ABSTRACT

Optimal linear estimators (OLEs) are designed for networked control systems (NCSs) with stochastic uncertainties, multiple sensors and multiple packet loss rates. Packet losses of both sides from sensors to an estimator (S–E) and from a controller to an actuator (C–A) are taken into account. A group of mutually uncorrelated stochastic variables obeying Bernoulli distributions are employed to depict the phenomenon of multiple packet losses from different S–E channels. The stochastic uncertainties in state and output matrices are depicted by white multiplicative noises. The OLEs dependent on the packet loss rates are presented in the least mean square (LMS) sense via the orthogonal projection approach (OPA) which is a universal and useful tool to obtain the optimal linear estimators in LMS sense. They are solved by three recursive equations including one Riccati equation, one Lyapunov equation and one simple difference equation. The stability of the OLEs is studied. A sufficient condition is provided to guarantee the steady-state property for time-invariant systems. Finally, a mass–spring–damper system is applied to confirm the performance of the derived algorithms.

1. Introduction

Recently, NCSs have been an interesting area of research [1,2]. In NCSs, there often exist packet losses in data transmissions of S–E and C–A due to unreliable communication channels. Therefore, it is significant how to design an estimator or a controller based on the incomplete data.

For NCSs with missing measurements, many algorithms about the design of estimators have been reported, e.g., the LMS filter [3], the covariance information based least-square (LS) estimator [4], the robust H_∞ filter [5], and the Kalman filter with intermittent observations [6]. For NCSs with packet losses of single side from S–E channel, the latest data arriving at the estimator will be applied to compensate the packet losses if the present packet is lost. Based on the simple compensation approach, an H_2 suboptimal filter is presented via a linear matrix inequality (LMI) method [7], the LMS optimal and steady-state linear estimators are presented via OPA [8], and the optimal full-order estimators are also developed via completing square method [9]. For NCSs with random delays of both S–E and C–A channels, the robust mixed H_2/H_∞ control problem is reported [10]. The robust H_2/H_∞ filtering problem is also studied for Takagi–Sugeno fuzzy model [11]. However, packet dropouts are not taken into account in [10,11]. For NCSs with packet losses of both S–E and C–A channels, the steady-state suboptimal H_2 prior filter via an LMI method [12] and the optimal linear filter via OPA [13] are designed, respectively. However, the prediction and smoothing problems are not taken into consideration. Further, for NCSs with data losses, delays and missing measurements, the adaptive suboptimal filter [14] and optimal linear estimators [15] are proposed via Riccati equation approach, respectively. The aforementioned references are mainly focused on systems measured by single sensor. However, there usually exist multiple sensors in many applications such as sensor networks. Recently, a robust weighted H_∞ filter was proposed for nonlinear networked systems with intermitted measurements [16]. The centralized and distributed information fusion estimators in the LMS sense [17] and LS sense [18,19] have been designed for systems with different loss rates. Moreover, the distributed fusion filter is proposed for systems with different delay and loss rates [20]. The centralized fusion estimators are also investigated for systems with multiple sensors subject to data losses, random delays and uncertain observations [21]. In a new recent literature [22], a robust weighted fusion predictor is designed for systems with known upper bounds of uncertain noise variances. However, the data losses from C–A channel are not taken into consideration in [16–22].

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In the references mentioned above, the coefficient matrices of systems considered are all deterministic. In practice, however, many systems are usually subject to external disturbances that make the uncertainties of systems [23]. For systems with stochastic uncertainties of multiplicative Gaussian or non-Gaussian noises, the polynomial state estimators are designed [24,25]. However, the designed filters are nonlinear and expensive in the computational cost. For the stochastic parameter systems, optimal linear filter [26] and distributed fusion filter [27] are derived, respectively. For stochastic uncertain systems subject to missing measurements, the recursive filter with correlated noises [28] and the quantized recursive filter [29] are studied for nonlinear stochastic system. The robust filters are designed [30] where the filtering gain is a stochastic matrix. The information fusion estimators for multi-sensor systems are also studied [31]. However, the multiple packet losses are not involved in the above-mentioned works. Recently, optimal linear estimators for a single-sensor system with multiplicative noises and packet losses from S–E channel are designed [32]. However, packet losses from C–A channel are not taken into consideration. In [33], a robust H_∞ filter is also investigated for systems with sector-bounded nonlinear and network constraints.

Multiple sensors, stochastic uncertainties and packet losses usually exist in NCSs simultaneously. It is significant to design the estimators to adapt these complicated cases simultaneously. So far, to the best of authors' knowledge, the optimal filtering problem for multi-sensor stochastic uncertain systems subject to multiple packet losses of both sides is not reported. In this paper, we consider the LMS recursive OLEs for NCSs with multiple sensors, stochastic uncertainties and packet losses. White multiplicative noises are introduced into the coefficient matrices to depict the stochastic uncertainty of systems. Packet losses randomly occur in data transmissions of both sides, and different communication channels possibly have the different packet loss rates. Using OPA [34], the LMS recursive OLEs are derived, which is involved in solving three difference equations including one Riccati, one Lyapunov and one simple difference. The stability and steady-state property are analyzed. The designed filter only depends on the data arrival rates of both sides. In the absence of multiplicative noises and packet losses, the proposed algorithm is reduced to the standard Kalman filter. Differently from the traditional estimators without stochastic uncertainties and packet losses, the covariance and gain matrices are affected by the control input, the state mean and second-order moment. Differently from the estimators in [31] where missing measurements of sensors are not compensated, that says that only noise is used when measurements are missing. However, in the current work, the latest measurement received previously is used for compensation when a packet is lost. Moreover, the C–A channel is not taken into account in the system of [31].

The main contribution of the current work is that we investigate the more comprehensive case: system is measured by multiple sensors; state and output matrices are subjected to the stochastic uncertainties of multiplicative noises, and data transmissions in both S–E and C–A channels have different packet dropout rates. The presented OLEs only use the arrival probabilities but not the sequence of received/lost indicators for every measurement. They can be calculated offline and have the steady-state property, which means the reduced online computational cost.

The outline of the current work is organized as follows. Section 2 gives the system model and problem statement. Section 3 derives the OLEs in the LMS sense. Section 4 is devoted to analyze the stability and steady-state property. Section 5 provides an example. Section 6 draws a conclusion. Finally, we provide the proofs of Lemma 2 and Theorems 1 and 2 in Appendices A, B and C.

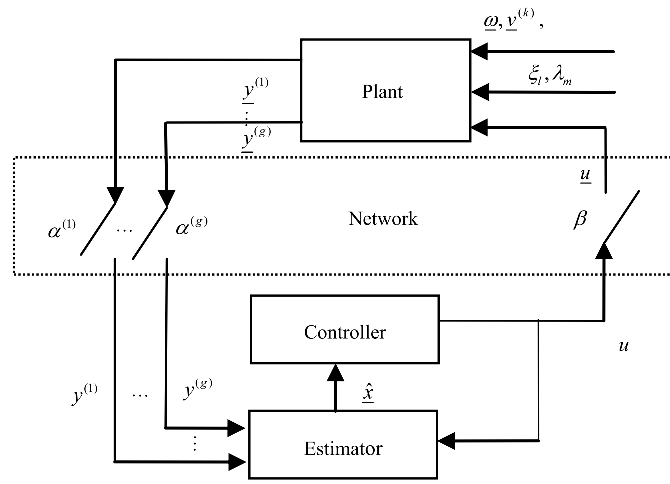

Fig. 1. NCS schematic with multiplicative noises, multiple sensors and multiple packet losses.

Notations. The following notational conventions will be used.

$\mathbb{R}^n$	n dimensional Euclidean space	I_n	n by n identity matrix
0	zero matrix with suitable dimension	diag(•)	diagonal matrix
E{•}	mathematical expectation operator	$\mathbf{1}_{m_k n_l}$	m_k by n_l matrix of all ones
Prob{•}	occurrence probability of event "•"	$\odot$	Hadamard product
$S_i = \{1, \cdots, i\}$	a subset of natural number whose maximum element is i	$\otimes$	Kronecker product
g	the number of sensors	A^{T}	transpose of matrix A
superscript (k)	the kth sensor	A^{-1}	inverse of matrix A
$\perp$	orthogonality	$\sigma(\bullet)$	spectrum radius of matrix •

2. Problem statement

The discrete stochastic linear system subject to multiplicative noises, multiple sensors and multiple packet losses can be formulated as (see Fig. 1):

$$\underline{x}_{t+1} = \left[\widehat{\Phi}_t + \sum_{l=1}^{\mu} \xi_{l,t} \widehat{\varphi}_{l,t}\right] \underline{x}_t + \widehat{F}_t \underline{u}_t + \widehat{\Gamma}_t \underline{\omega}_t \tag{1}$$

$$\underline{y}_t^{(k)} = \left[\widehat{H}_t^{(k)} + \sum_{m=1}^{\rho^{(k)}} \lambda_{m,t}^{(k)} \widehat{h}_{m,t}^{(k)}\right] \underline{x}_t + \widehat{G}_t^{(k)} \underline{u}_t + \underline{v}_t^{(k)}, \quad k \in S_g \tag{2}$$

$$y_t^{(k)} = \alpha_t^{(k)} \underline{y}_t^{(k)} + \left(1 - \alpha_t^{(k)}\right) y_{t-1}^{(k)}, \quad k \in S_g \tag{3}$$

$$\underline{u}_t = \beta_t u_t + (1 - \beta_t) \underline{u}_{t-1} \tag{4}$$

where $\underline{x}_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^{n_u}$, $\underline{u}_t \in \mathbb{R}^{n_u}$, $\underline{y}_t^{(k)} \in \mathbb{R}^{n_y^{(k)}}$, $y_t^{(k)} \in \mathbb{R}^{n_y^{(k)}}$, $\underline{\omega}_t \in \mathbb{R}^{n_\omega}$ and $\underline{v}_t^{(k)} \in \mathbb{R}^{n_y^{(k)}}$ represent the system state, the known control input to be sent to the actuator, the control input received by the actuator, the observation outputs to be sent to the estimator, the observations received by the estimator, the process noise and the observation noises, respectively. The integer $t \geq 0$ is the tth sampling time. Multiplicative noises $\xi_{l,t}$, $l \in S_\mu$ and $\lambda_{m,t}^{(k)}$, $m \in S_{\rho^{(k)}}$ are mutually uncorrelated scalar white noises with mean zeros and variance