Mechanical Systems and Signal Processing 80 (2016) 245–261

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Point mass identification in rectangular plates from minimal natural frequency data

Lourdes Rubio [a], José Fernández-Sáez [b], Antonino Morassi [c,*]

[a] Department of Mechanical Engineering, University Carlos III of Madrid, Avda. de la Universidad 30, 28911 Leganés, Madrid, Spain
[b] Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Avda. de la Universidad 30, 28911 Leganés, Madrid, Spain
[c] Dipartimento Politecnico di Ingegneria e Architettura, Università degli Studi di Udine, via Cotonificio 114, 33100 Udine, Italy

ARTICLE INFO

Article history:
Received 4 August 2015
Received in revised form
20 April 2016
Accepted 22 April 2016
Available online 2 May 2016

Keywords:
Point mass
Plates
Natural frequencies
Inverse problems

ABSTRACT

The inverse problem of determining the location and size of a point mass attached on a simply supported, isotropic and homogeneous rectangular plate from minimal natural frequency data is considered in this paper. Under the assumption that the size of the mass is small compared to the total mass of the plate, we show that the problem can be formulated and solved in closed form in terms of point mass-induced changes on the first three natural frequencies. Numerical simulations indicate that the method allows for accurate identification, provided that measurement/modelling errors are smaller than eigenfrequency changes.

1. Introduction

The development of effective methods for timely identification of concentrated masses in vibrating plates is an issue of increasing interest in several fields of technology, such as, for instance, slabs supporting engines or motors not directly accessible from the exterior; printed circuits boards or plate-like chassis with electronic elements attached to them; plates with inhomogeneities in material mass density due to defective manufacturing processes; see, for example, the recent paper by Aydogdu and Filiz [1] and references cited therein. Moreover, in connection with the above applications, we also recall the possibility of using concentrated masses to modify the resonant frequencies of a thin uniform rectangular plate [2]. Ostachowicz et al. considered in [3] the interesting inverse problem of determining the location and size of a concentrated mass in a rectangular plate from natural frequency measurements. The authors used a class of optimization methods based on a genetic algorithm for solving the mass identification problem. The procedure was demonstrated on a simply supported rectangular plate made by linearly elastic isotropic and homogeneous material, and the first four natural frequencies were considered to construct the objective/error function to be minimized. In this paper we re-examine the problem by Ostachowicz et al. and, under the assumption that the size of the concentrated mass is small compared with the global mass of the plate, we derive a closed form solution based on natural frequency data. In particular, following a line of research on detection of defects in rods and beams (see, for instance, [4]), we carry out an identification based on a *minimal* set of natural frequencies. Since the unperturbed plate, as in [3], is completely known and a single point mass is attached, only three parameters need to be determined, namely the size of the mass and the two Cartesian coordinates of its location.

* Corresponding author.
E-mail addresses: lrubio@ing.uc3m.es (L. Rubio), ppfer@ing.uc3m.es (J. Fernández-Sáez), antonino.morassi@uniud.it (A. Morassi).

http://dx.doi.org/10.1016/j.ymssp.2016.04.025
0888-3270/

Therefore, we investigate to what extent the measurement of the point mass-induced changes in the first three natural frequencies can be useful for mass identification.

Changes in natural frequencies are commonly used as concentrated mass indicators, since these data are relatively easy to measure with a satisfactory level of accuracy. However, the inverse problem formulated in terms of natural frequencies is intrinsically ill-posed, since, by the double symmetry of the unperturbed plate (i.e., the plate without point mass), a point mass located at any of a set of symmetrically placed points will produce identical changes in natural frequencies. This feature is confirmed by the symmetry of the objective function with respect to the mid-point axes of the rectangular plate found in [3]. Despite this ill-posedness of the inverse problem, we show that the linearized inverse problem can be solved in closed form from the knowledge of the changes in the first three natural frequencies, and the point mass can be uniquely localized up to a double symmetry location with respect to mid-point axes of the rectangular domain. The analysis also shows that a slightly different approach must be followed when the second eigenvalue of the unperturbed plate has double multiplicity, that is, for a square plate.

Our method is essentially based on the determination of an explicit expression of the sensitivity of a natural frequency to a single point mass (i.e., the first derivative with respect to the point mass size), and on the simple form that this expression takes in the case of a uniform, simply supported rectangular plate. The expression of the first derivative can be obtained for general domains and inhomogeneous anisotropic materials, following the lines of the classical perturbation theory of eigenvalues, see, for example, [5] and [6]. In particular, the analysis of the multiple eigenvalue case allowed us to give a theoretical justification of the interesting phenomenon observed, both numerically and experimentally, by Amabili et al. in [7], namely, the fact that a small mass placed on the diagonal of a square plate is enough to transform the shape of modes ($m = 1, n = 2$) and ($m = 2, n = 1$), where m and n are the number of half-waves in the two directions parallel to the plate sides, into vibrating modes with diagonal nodal lines.

The identification method has been tested on a series of numerical simulations. Numerical results support the theory in the absence of noise on the natural frequency data. The possible presence of measurement errors will result in a loss of accuracy in predicting the mass location and intensity, especially when the point mass is close to the boundary of the plate. The paper is organized as follows. The frequency sensitivity to a point mass is derived in Section 2 and it is specialized to simply supported rectangular plates in Section 3. The identification method is presented in Section 4. A selected set of results of numerical simulations is collected in Section 5.

2. Eigenpair sensitivity to a point mass

2.1. Formulation of the problem

Free transversal infinitesimal vibrations with radian frequency ω of a thin elastic clamped plate are governed by the boundary value problem

$$\begin{cases} M_{\alpha\beta,\alpha\beta}(u) + \lambda\rho u = 0, & \text{in } \Omega, \quad \text{(a)} \\ u = \dfrac{\partial u}{\partial n} = 0, & \text{on } \partial\Omega, \quad \text{(b)} \end{cases} \tag{1}$$

where Ω is a regular bounded domain in $\mathbb{R}^2$ coinciding with the middle-plane of the plate, n is the unit outer normal to $\partial\Omega$ and $u \in H_0^2(\Omega)$ describes the eigenfunction associated to the eigenvalue $\lambda = \omega^2$. Here, $H_0^2(\Omega)$ is the Hilbert space of the functions $f: \Omega \to \mathbb{R}$ such that f, the first and the second weak gradient of f are square summable on Ω, that is $\int_\Omega (f^2 + |\nabla f|^2 + |\nabla^2 f|^2) < \infty$, and the trace of f and of the gradient of f vanish on the boundary $\partial\Omega$. Moreover, hereinafter, repeated Greek indexes are assumed to be summed from 1 to 2. The plate is assumed to have no material damping, since its effect on the natural frequencies is known to be negligible. The quantities $M_{\alpha\beta}(u) = -P_{\alpha\beta\gamma\delta} u_{,\gamma\delta}$, $\alpha, \beta, \gamma, \delta = 1, 2$, and ρ denote the bending/torsional moments in a reference Cartesian system (O, X_1, X_2) and the mass density for unit area (of the middle plane) of the plate, respectively. The coefficients $P_{\alpha\beta\gamma\delta} = \frac{h^3}{12} C_{\alpha\beta\gamma\delta}$, $\alpha, \beta, \gamma, \delta = 1, 2$, are the Cartesian components of the plate tensor $\mathbb{P}$, where $\mathbb{C}$ is the elasticity tensor of the material and h is the uniform plate thickness. We shall be concerned with plates for which $\mathbb{P}$ satisfies:

(i) the minor and major symmetries, i.e., $P_{\alpha\beta\gamma\delta} = P_{\beta\alpha\gamma\delta} = P_{\alpha\beta\delta\gamma}$ and $P_{\alpha\beta\gamma\delta} = P_{\gamma\delta\alpha\beta}$;
(ii) $\mathbb{P}$ is a strongly convex fourth order tensor, i.e., $\mathbb{P}A \cdot A \ge \xi |A|^2$ in Ω for every 2×2 real symmetric matrix A, where $\xi > 0$ is a constant and $\cdot$ is the usual scalar product between second order tensors;
(iii) $\mathbb{P}$ belongs to $C^2(\overline{\Omega})$.

The function ρ will be assumed to be a continuous and strictly positive on $\overline{\Omega}$.

Vibration modes and corresponding natural frequencies are the eigensolutions of the boundary value problem (1a)–(1b), and we denote by $(u_n, \lambda_n = \omega_n^2)$, $n \ge 1$, the nth eigenpair of the *unperturbed* plate, that is the plate without the point mass. It is well-known that for such $\mathbb{P}$ and ρ, and under our assumptions on Ω, there exists an infinite sequence $\{\lambda_n\}_{n=1}^{\infty}$ of eigenvalues of (1a) and (1b) such that $0 < \lambda_1 \le \lambda_2 \le \lambda_3 \le \ldots$, with $\lim_{n\to\infty} \lambda_n = \infty$; see [5]. Assume now that a point mass ϵ is attached at a