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Application of Cauchy wavelet transformation to identify time-variant modal parameters of structures

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ABSTRACT

This work proposes a procedure for accurately identifying instantaneous modal parameters of a linear time-varying system using a time-varying autoregressive with exogenous input (TVARX) model with the continuous Cauchy wavelet transform (CCWT). An appropriate TVARX model is established using the velocity and displacement responses of the system under consideration. The time-varying coefficients of the TVARX are expanded as piecewise polynomial functions. CCWTs with various scale parameters are then applied to the TVARX model to evaluate the instantaneous modal parameters of different modes. The CCWTs of the velocity and displacement responses are analytically obtained from the CCWT of the measured acceleration responses. The effectiveness and accuracy of the proposed procedure are validated by numerical simulations of single and multiple degrees of freedom systems that have periodically varying and sharply varying stiffness and damping coefficients. The effects of noise, the Cauchy wavelet function and the order of the polynomial on the evaluation of the modal parameters are explored in processing the numerically simulated acceleration responses of systems with a single degree of freedom subjected to base excitation. Finally, the proposed procedure is adopted to determine the modal parameters of a five-story symmetric steel frame from its measured acceleration responses in a shaking table test. The measured strains reveal the yielding of columns in the first story. The variations of the identified instantaneous natural frequencies and modal damping ratios with time are consistent with the physical phenomena that are observed from the measured strains and base excitation acceleration.

1. Introduction

Monitoring of the health of a structure can detect its degradation and prevent catastrophic failure. Among available approaches to so doing, data driven vibration-based structural damage detection methods are popular owing to their simplicity, and competitive costs of instrumentation and data analysis [1–6]. Such methods typically compare the current characteristic features of a structure with those for a state without damage to detect the possible damages in the structure. The characteristic structural features are usually determined from the modal frequencies, damping ratios and modal shapes, which are extracted from the measured dynamic responses of the structure. However, the identified modal parameters can be significantly affected by the environment, including, for example, temperature [7,8]. To eliminate the environmental

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effects, vibration-based structural damage detection methods can utilize the time-variant modal parameters that are evaluated from the dynamic responses of a structure to a severe event, such as a strong earthquake. If a structure is damaged under dynamic loading, then its stiffness and damping ratios are likely to change accordingly, causing its modal parameters to vary over time.

The time-varying autoregressive with exogenous input (TVARX) model is normally adopted to establish an input-output relationship between the dynamic responses and input forces of a time-varying linear system; it has important applications in various fields [9–12]. The time-variant modal parameters of the system can be extracted from the coefficients of such a model [13,14]. TVARX models are typically constructed using a recursive least-squares approach or by the basis function expansion with regression approach.

The recursive least-squares approach is an on-line method with high computational efficiency, but is slow in tracking time-varying coefficients of a TVARX model and is highly sensitive to initial conditions and noise [9,15]. A number of techniques have been developed to improve the recursive least-squares approach in tracking time-varying coefficients. They include the use of variable forgetting factors [16,17], covariance matrix resetting [18,19], the sliding window technique [20,21], the use of the Kalman filter [13] and the use of the random walk Kalman filter [22].

The basis function expansion with regression approach generally has the advantage of excellently tracking time-varying coefficients in the TVARX model. Numerous sets of functions have been chosen as basis functions, including regular polynomials [10], Legendre polynomials [23,24], shape functions that are constructed by a moving least-squares approach with regular low-order polynomials [14,25], Fourier series [26], Walsh functions [24,27], radial basis functions [28] and wavelets [29–32]. Selecting the proper basis functions can be crucial to the success of the basis function expansion with regression approach. Zou et al. [27] numerically demonstrated that the Legendre polynomials are suitable for coefficients that change smoothly over time, whereas Walsh functions are effective for piecewise stationary time-varying coefficients. Asutkar et al. [31] established that Haar basis functions are better than Walsh, cosine and Legendre basis functions in constructing the step-function-type coefficient functions in TVARX model. To reduce difficulties in the selection of proper basis functions, Li et al. [32] combined cardinal B-spline wavelets with a block least mean square or orthogonal least square algorithm to track effectively both rapid and slow changes of coefficients with time. Most of the aforementioned works demonstrated the efficiency of the proposed methods in accurately determining the coefficients in TVARX models by processing numerically simulated data from known TVARX models. Only Huang et al. [14] and Su et al. [25] validated their approaches by processing numerically simulated responses by solving equations of motion for time-varying linear systems and identifying the corresponding instantaneous modal parameters; they thereby demonstrated the advantages of their scheme over a recursive technique with a forgetting factor and a weighted basis function expansion approach on tracking the variations of instantaneous modal parameters.

There are many methods other than the aforementioned two classes of approaches for identifying time-dependent parameters of a time-varying linear system. Some different techniques were proposed recently. For example, Dziedziech et al. [33] utilized wavelet-based frequency response functions to determine modal parameters of a time-varying linear system, while Zhou et al. [34] proposed a two-stage least squares method for the output-only case. Louarroudi et al. [35] presented a frequency domain method for a periodically time-varying system with single-input single-output. Zhou et al. [36] showed a time–frequency-domain maximum likelihood approach for the output-only case with the assumption of white-noise inputs. To perform output-only modal analyses, Yang et al. [37] proposed the functional series time-dependent autoregressive models using Kriging shape functions as basis functions, while Spiridonakos and Fassois [38] provided a critical survey on the applications of the functional series time-dependent autoregressive moving average models.

Huang et al. [14] mathematically proved that accurate instantaneous modal parameters of a time-varying linear system can be obtained from an appropriate TVARX model that is based on the displacement responses of the system and a TVARX model that is developed using acceleration or velocity responses can cause significant systematic errors in the estimation of modal parameters. However, in real applications, and especially for the monitoring of structures in an earthquake, acceleration or velocity responses are typically measured. A good scheme is needed to obtain displacement responses from measured acceleration or velocity responses.

An excellent signal processing technique called wavelet transform has been theoretically developed over the last two decades. Wavelet transformations support time-frequency analyses by presenting information in the time and frequency domains simultaneously. Various wavelet functions (Haar wavelets, Meyer wavelets, Morlet wavelets, Daubechies wavelets, Cauchy wavelets, B-spline wavelets and others) and different wavelet transforms (discrete wavelet transform, wavelet packet transform, continuous wavelet transform, stationary wavelet transform, and stationary wavelet packet transform) have been proposed and successfully applied in many fields [39–43]. Among existing wavelet functions and wavelet transformations, the continuous Cauchy wavelet transform (CCWT) provides analytical relations between the transforms of a function and its derivatives. Such relations are very helpful in developing a TVARX model that corresponds to displacement and velocity responses from measured acceleration data.

Few studies have employed the CCWT to extract modal parameters from the dynamic responses of a structure. Argoul and Le [44] proposed four instantaneous indicators that were based on the ridges and skeletons that were extracted from the CCWT of transient responses of a structure to characterize its nonlinear behaviors. Le and Argoul [45] applied the continuous wavelet transform to the free-decay responses of a linear system with weak damping using various wavelet