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# Nonlinear modeling and dynamic analysis of a hydro-turbine governing system in the process of sudden load increase transient

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ABSTRACT

In order to study the nonlinear dynamic behaviors of a hydro-turbine governing system in the process of sudden load increase transient, we establish a novel nonlinear dynamic model of the hydro-turbine governing system which considers the elastic water-hammer model of the penstock and the second-order model of the generator. The six nonlinear dynamic transfer coefficients of the hydro-turbine are innovatively proposed by utilizing internal characteristics and analyzing the change laws of the characteristic parameters of the hydro-turbine governing system. Moreover, from the point of view of engineering, the nonlinear dynamic behaviors of the above system are exhaustively investigated based on bifurcation diagrams and time waveforms. More importantly, all of the above analyses supply theoretical basis for allowing a hydropower station to maintain a stable operation in the process of sudden load increase transient.



## 1. Introduction

A hydro-turbine governing system (HTGS) is a complex nonlinear system, which is composed of synchronous generator, hydro-turbine, governor and penstock [1–3]. The dynamic quality of the HTGS is directly related to the safety, stability and economical operation of the hydropower station [4–8]. In recent years, with the development of hydroelectric power industry, a large number of hydropower stations with high water-head and large single unit have been built in China, which leads to the unstable problem of the HTGS [9–14]. Therefore, for the stable operation of the hydropower station, the study of the complex nonlinear dynamic behaviors of the HTGS has important theoretical significance and practical value.

Transition process is divided into large fluctuation process and small fluctuation process. In previous studies, the investigation of the transition process of the HTGS mainly focuses on the process of load rejection transient, which use linear model to indicate the dynamic behaviors of the HTGS in the small fluctuation process [4–6,12]. Whereas, the changes of system parameters affect the stability of the governing system, even these dynamical changes are very small [4,15–24]. Apparently, this kind of simplified model is not enough to describe the complex dynamic characteristics of the HTGS. Also, for the large fluctuation process, the method of characteristic indicator of the hydro-turbine and the form of polynomial fitting are widely used in many published papers to investigate the relationship of characteristic parameters of the HTGS

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**Nomenclature**

| | |
|---|---|
| $M_t$ | the mechanical torque of the hydro-turbine, N m |
| $H$ | the hydro-turbine head, m |
| $n$ | the hydro-turbine speed, rad/s |
| $Q$ | the hydro-turbine flow, $m^3/s$ |
| $m_t$ | the deviation of the mechanical torque of the hydro-turbine, p.u. |
| $h$ | the deviation of the hydro-turbine head, p u. |
| $x$ | the deviation of the hydro-turbine speed, p u. |
| $q$ | the deviation of the hydro-turbine flow, p u. |
| $\delta$ | the rotor angle, rad |
| $y$ | the deviation of the guide vane opening, p u. |
| $\omega$ | the deviation of the generator rotor speed, p.u. |
| $W$ | the generator rotor speed, rad/s |
| $h_w$ | the characteristic coefficient of the pipeline |
| $T_{ab}$ | the mechanical starting time, $s$ |
| $T_y$ | the engager relay time constant, $s$ |
| $u$ | the output of the regulator |
| $\beta$ | the normal angle of the guide vane, rad |
| $\eta$ | the efficiency of the hydro-turbine, % |
| $E_q'$ | the transient internal voltage of armature |
| $V_s$ | the voltage of infinite bus |
| $x_d'$ | the direct axis transient reactance |
| $x_q$ | the quartered axis reactance |
| $\alpha$ | the guide vane discharge angle, rad |
| $b_t$ | the height of guide vane, m |
| $F$ | the runner outlet area, $m^2$ |
| $\beta_0$ | the runner intermediate flow surface angle, rad |
| $r_0$ | the runner intermediate flow surface radius, m |
| $D_0$ | the guide vane pitch circle diameter, m |
| $L$ | the width of the guide vane, m |
| $Z_0$ | the number of the guide vane |
| $Y$ | the guide vane opening, rad |
| $D_1$ | the runner diameter of the hydro-turbine, m |
| $e_{mx}$,$e_{my}$,$e_{mh}$ | the partial derivatives of the hydro-turbine torque with respect to the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head, p.u. |
| $e_{qx}$,$e_{qy}$,$e_{qh}$ | the partial derivatives of the flow with respect to the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head, p.u. |
| $k_p$ | the proportional adjustment coefficient |
| $k_i$ | the integral adjustment coefficient |
| $k_d$ | the differential adjustment coefficient |
| $Y_k$ | the main servomotor stroke |
| $d$ | the flow passage component |

[23–27]. However, the mathematical models established by the above methods are not convenient to deeply analyze the dynamic characteristics of the HTGS [25–27]. Therefore, building a proper mathematical model becomes a necessary and difficult work.

Motivated by the above discussions, we have three advantages which make our approach attractive, comparing with the prior work. First, we creatively present six nonlinear dynamic transfer coefficients of the hydro-turbine. Second, a novel nonlinear dynamic mathematical model of the HTGS is established in the process of sudden load increase transient. Third, concentrating on the engineering significance, we investigate the nonlinear dynamic behaviors of the above system in detail.

The rest of the paper is organized as follows: In Section 2, a novel nonlinear dynamic mathematical model of the HTGS is proposed in the process of sudden load increase transient. We analyze the nonlinear dynamic behaviors of the established model in Section 3. Conclusions and discussion in Section 4 close the paper.

## 2. A nonlinear dynamic model of the HTGS

### 2.1. Mathematical model of the Francis hydro-turbine

The structure of the Francis hydro-turbine governing system [20] is shown in Fig. 1.

The dynamic characteristics of a Francis hydro-turbine [6,20] can be described as

$$\begin{cases} M_t = M_t(H, n, Y) \\ Q = Q(H, n, Y) \end{cases}, \tag{1}$$

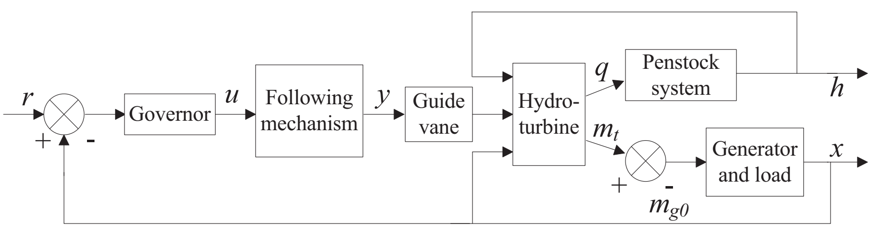


**Fig. 1.** The structure diagram of the Francis hydro-turbine governing system.