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Optimal phase calibration of nonlinear, delayed sensors

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ABSTRACT

Sensor calibration is a routine task which is often performed under the assumption of linearity and immediate response. The present paper addresses the task of calibrating a statically or zero-memory nonlinear sensor given delayed measurements that can give rise to a multi-valued relationship. A simple, optimal, non-parametric figure-of-merit is proposed to eliminate the delay or phase lag in sensing without the use of parametric models or Fourier transformation. The phase estimation at a selected frequency is not accurate when some nonlinear distortions are present. It is shown that a delayed measurement of a calibration device under periodic oscillations, creates a Lissajous-like curve which encloses an area directly proportional to the delay time. An efficient numerical optimization based on Green's integral, the time-shift of the reference sensor is varied until a non-delayed, single-valued calibration curve is obtained.

1. Introduction

The present paper introduces a time or phase delay identification method that is insensitive to zero-memory nonlinear distortions [1]. Many modern sensors rely on internal analog and digital electronics which introduce some delay to the output signal. Delay is also introduced by the physical nature of the sensing device, e.g., optical, magnetic or capacitive [2]. Indeed, phase delay measurements play an important role in some sensors, e.g., Coriolis flow meters [2–4]. Calibration is often employed to overcome some nonlinear behavior that a real system can exhibit. Any time delay caused by the measurement system needs to be pre-calibrated to improve accuracy [5]. In order to estimate the phase or time delay, Fourier analysis [3,4] is often employed and the phase difference is estimated at a single frequency at which the measured structure is excited. It is shown here that slight harmonic distortion on the measured surface and the calibrated sensor, introduce an error that the simple Fourier analysis cannot eliminate. Rather than embarking upon a nonlinear identification and curve-fitting based parameter estimation, a non-parametric delay estimation method is put forward. The method is independent of the type of nonlinear distortions, as long as the sensor exhibits zero-memory type of nonlinearity.

Phase measurement using Lissajous curves is an old technique [6], routinely employed using oscilloscopes and data recorders. Monochromatic signals produce an ellipse whose geometrical properties are related to the amplitude and phase ratios [7]. Any nonlinear behavior of the measurement system results in a response with multiple frequencies thus giving rise to more complex Lissajous-type, possibly closed, curves. These curves do entrap some area inside, and can exhibit self-intersections or knots [8,9] when the sensor's calibration curve is not one-to-one. Since Lissajous figures are being used routinely to measure the phase between two monochromatic signals [6], they are used in medical and physical applications to assess the phase between two signals [10]. Indeed, for a single frequency one usually obtains an ellipse in the calibration plane, but when more complicated signals are involved, Fourier based methods are some time employed [11,12]. Multi-chromatic signals are more complex to decipher [8] than monochromatic ones and neither Fourier or ellipse-fit [7,13] are suitable for nonlinearly distorted signals,

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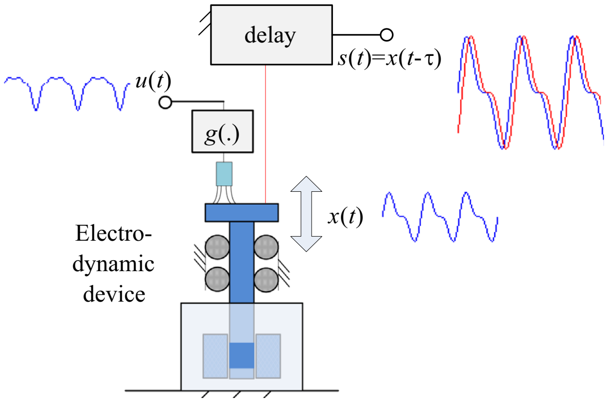

Fig. 1. Calibration system showing a delayed sensor, $s(t)=x(t-\tau)$, a nonlinear sensor, $u(t)$ both measuring the motion of the surface, $x(t)$.

as will be demonstrated. The present paper seeks to bypass model-based curve-fitting by proposing a closed form computation of a merit function whose minimization produces the desired time-delay.

The paper begins by introducing the sensor calibration problem, followed by a brief mathematical explanation of the inaccuracy caused by harmonic distortions using Fourier analysis. Later, the basis for the proposed method is explained and the connection between closed Lissajous curve, the area they encompass and the time/phase delay is developed using a graphical example and mathematically through a close form expression of the delay. Later, the computational aspects using digital computation are shown and finally several numerical and experimental examples are provided.

1.1. *Description of the problem*

Consider the system described in Fig. 1 with an oscillating surface whose motion is $x(t)$, the reference sensor introducing delayed output $s(t-\tau)$ and the nonlinear sensor to be calibrated produces $u(t)=g(x(t))$. The main difficulty arises from the nonlinear mapping $g(\cdot)$ which couples different frequencies and spectral lines in an inseparable manner. Clearly, the problem does not exist when all sensors are linear and when the measured surface oscillates at a single frequency, as will be proved later. An elaborate alternative to the proposed method involves curve fitting the measured response, $x(t)$, in advance, identifying the nonlinear mapping of the sensor $g(\cdot)$ and use these to compensate for the nonlinear distortions. The latter is an elaborate exercise that poses numerous difficulties and will not be discussed any further here. Indeed, the proposed approach provides a direct visualization of the time-delay between the reference and calibrated sensors, with no need to curve-fit or construct a parametric model.

In order to illustrate the deficiency of Fourier based estimation in this case, an introductory example is presented alongside some basic mathematical preliminaries.

1.2. *Estimating the phase difference and the effect of harmonic distortions*

Consider a periodic oscillatory motion with small harmonic distortion indicated by α:

$$x(t)=X_0\ \sin\ \Omega t+\alpha\ \sin\ 2\Omega t. \tag{1}$$

The zero-memory nonlinear behavior of the calibrated sensor is approximated by a polynomial for sake of simplicity (although it has no effect on the algorithm). It would thus produce:

$$u(t)=g(x(t))\approx\sum_{n=0}^{N}b_n x^n(t) \tag{2}$$

The former signal's phase is compared with the delayed reference sensor's output:

$$s(t)=x(t-\tau)=X_0\ \sin\ \Omega(t-\tau)+\alpha\ \sin\ 2\Omega(t-\tau). \tag{3}$$

In order to obtain the unwanted phase shift between the sensors, two methods are examined. The first is a Fourier-based estimate and the second is the proposed algorithm.

1.3. *Phase estimation using Fourier analysis*

Fourier analysis computes the complex amplitude at each frequency. Here, $\Omega=1$ is chosen for simplicity without loss of generality (time can be scaled). Considering a periodic motion and zero-memory nonlinearity, one can compute the complex amplitudes from which the gain and phase can be obtained. In the example we consider powers up to $N=3$ in (2),