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Generalized fuzzy *b*-closed and generalized ⋆-fuzzy *b*-closed sets in double fuzzy topological spaces

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ABSTRACT

The purpose of this paper is to introduce and study a new class of fuzzy sets called (r, s)-generalized fuzzy *b*-closed sets and (r, s)-generalized ⋆-fuzzy *b*-closed sets in double fuzzy topological spaces. Furthermore, the relationships between the new concepts are introduced and established with some interesting examples.

1. Introduction

A progressive development of fuzzy sets [1] has been made to discover the fuzzy analogues of the crisp sets theory. On the other hand, the idea of intuitionistic fuzzy sets was first introduced by Atanassov [2]. Later on, Çoker [3] presented the notion of intuitionistic fuzzy topology. Samanta and Mondal [4], introduced and characterized the intuitionistic gradation of openness of fuzzy sets which is a generalization of smooth topology and the topology of intuitionistic fuzzy sets. The name "intuitionistic" is discontinued in mathematics and applications. Garcia and Rodabaugh [5] concluded that they work under the name "double".

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In 2009, Omari and Noorani [6] introduced generalized b-closed sets (briefly, gb-closed) in general topology. As a generalization of the results in References 6 and 7, we introduce and study (r, s)-generalized fuzzy b-closed sets in double fuzzy topological spaces, then a new class of fuzzy sets between an (r, s)-fuzzy b-closed sets and an (r, s)-generalized fuzzy b-closed sets namely (r, s)-generalized $\star$-fuzzy b-closed sets is introduced and investigated. Finally, the relationships between (r, s)-generalized fuzzy b-closed and (r, s)-generalized $\star$-fuzzy b-closed sets are introduced and established with some interesting counter examples.

2. Preliminaries

Throughout this paper, X will be a non-empty set, $I = [0, 1]$, $I_0 = (0, 1]$ and $I_1 = [0, 1)$. A fuzzy set λ is quasi-coincident with a fuzzy set μ (denoted by, $\lambda q \mu$) iff there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$ and they are not quasi-coincident otherwise (denoted by, $\lambda \bar{q} \mu$). The family of all fuzzy sets on X is denoted by I^X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $\lambda \in I^X$, $\underline{1} - \lambda$ denotes its complement. All other notations are standard notations of fuzzy set theory.

Now, we recall the following definitions which are useful in the sequel.

Definition 2.1. (see [4]) A double fuzzy topology (τ, τ^*) on X is a pair of maps $\tau, \tau^* : I^X \to I$, which satisfies the following properties:

(O1) $\tau(\lambda) \le \underline{1} - \tau^*(\lambda)$ for each $\lambda \in I^X$.

(O2) $\tau(\lambda_1 \wedge \lambda_2) \ge \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \le \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each λ_1, $\lambda_2 \in I^X$.

(O3) $\tau(\vee_{i\in\Gamma} \lambda_i) \ge \wedge_{i\in\Gamma} \tau(\lambda_i)$ and $\tau^*(\vee_{i\in\Gamma} \lambda_i) \le \vee_{i\in\Gamma} \tau^*(\lambda_i)$ for each $\lambda_i \in I^X$, $i \in \Gamma$.

The triplet (X, τ, τ^*) is called a double fuzzy topological space (briefly, dfts). A fuzzy set λ is called an (r, s)-fuzzy open (briefly, (r, s)-fo) if $\tau(\lambda) \ge r$ and $\tau^*(\lambda) \le s$. A fuzzy set λ is called an (r, s)-fuzzy closed (briefly, (r, s)-fc) set iff $\underline{1} - \lambda$ is an (r, s)-fo set.

Theorem 2.1. (see [8]) Let (X, τ, τ^*) be a dfts. Then double fuzzy closure operator and double fuzzy interior operator of $\lambda \in I^X$ are defined by

$$C_{\tau,\tau^*}(\lambda, r, s) = \wedge\{\mu \in I^X | \lambda \le \mu, \tau(\underline{1} - \mu) \ge r, \tau^*(\underline{1} - \mu) \le s\},$$

$$I_{\tau,\tau^*}(\lambda, r, s) = \vee\{\mu \in I^X | \mu \le \lambda, \tau(\mu) \ge r, \tau^*(\mu) \le s\}.$$

Where $r \in I_0$ and $s \in I_1$ such that $r + s \le 1$.

Definition 2.2. Let (X, τ, τ^*) be a dfts. For each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$. A fuzzy set λ is called:

1. An (r, s)-fuzzy semiopen (see [9]) (briefly, (r, s)-fso) if $\lambda \le C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s)$. λ is called an (r, s)-fuzzy semi closed (briefly, (r, s)-fsc) iff $\underline{1} - \lambda$ is an (r, s)-fso set.
2. An (r, s)-generalized fuzzy closed (see [10]) (briefly, (r, s)-gfc) if $C_{\tau,\tau^*}(\lambda, r, s) \le \mu$, $\lambda \le \mu$, $\tau(\mu) \ge r$ and $\tau^*(\mu) \le s$. λ is called an (r, s)-generalized fuzzy open (briefly, (r, s)-gfo) iff $\underline{1} - \lambda$ is (r, s)-gfc set.

Definition 2.3. (see [11,12]) Let (X, τ, τ^*) be a dfts. For each λ, $\mu \in I^X$ and $r \in I_0$, $s \in I_1$. Then, a fuzzy set λ is said to be (r, s)-fuzzy generalized $\psi\rho$-closed (briefly, (r, s)-fg$\psi\rho$-closed) if $\psi C_{\tau,\tau^*}(\lambda, r, s) \le \mu$ such that $\lambda \le \mu$ and μ is (r, s)-fuzzy ρ-open set. λ is called (r, s)-fuzzy generalized $\psi\rho$-open (briefly, (r, s)-fg$\psi\rho$-open) iff $\underline{1} - \lambda$ is (r, s)-fg$\psi\rho$-closed set.

3. (r, s)-generalized fuzzy b-closed sets

In this section, we introduce and study some basic properties of a new class of fuzzy sets called an (r, s)-fuzzy b-closed sets and an (r, s)-generalized fuzzy b-closed.

Definition 3.1. Let (X, τ, τ^*) be a dfts. For each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$. A fuzzy set λ is called:

1. An (r, s)-fuzzy b-closed (briefly, (r, s)-fbc) if

$$\lambda \ge (I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s)) \wedge (C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s)).$$

λ is called an (r, s)-fuzzy b-open (briefly, (r, s)-fbo) iff $\underline{1} - \lambda$ is (r, s)-fbc set.

2. An (r, s)-generalized fuzzy b-closed (briefly, (r, s)-gfbc) if $bC_{\tau,\tau^*}(\lambda, r, s) \le \mu$, $\lambda \le \mu$, $\tau(\mu) \ge r$ and $\tau^*(\mu) \le s$. λ is called an (r, s)-generalized fuzzy b-open (briefly, (r, s)-gfbo) iff $\underline{1} - \lambda$ is (r, s)-gfbc set.

Definition 3.2. Let (X, τ, τ^*) be a dfts. Then double fuzzy b-closure operator and double fuzzy b-interior operator of $\lambda \in I^X$ are defined by

$$bC_{\tau,\tau^*}(\lambda, r, s) = \wedge\{\mu \in I^X | \lambda \le \mu \text{ and } \mu \text{ is } (r, s)\text{-fbc}\},$$

$$bI_{\tau,\tau^*}(\lambda, r, s) = \vee\{\mu \in I^X | \mu \le \lambda \text{ and } \mu \text{ is } (r, s)\text{-fbo}\}.$$

Where $r \in I_0$ and $s \in I_1$ such that $r + s \le 1$.

Remark 3.1. Every (r, s)-fbc set is an (r, s)-gfbc set.

The converse of the above remark may be not true as shown by the following example.

Example 3.1. Let $X = \{a, b\}$. Defined μ, α and β by:

$\mu(a) = 0.3, \quad \mu(b) = 0.4,$

$\alpha(a) = 0.4, \quad \alpha(b) = 0.5,$

$\beta(a) = 0.3, \quad \beta(b) = 0.7,$

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise.} \end{cases}$$