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Discontinuity preserving method for noise removal of multi-carrier signals

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ABSTRACT

A nonlinear method based on anisotropic diffusion notion is proposed in this paper to remove noise from noisy signals modulated with multiple carrier signals by preserving carrier signals as well as discontinuities present in the original noiseless signals. Gaussian and correlated noise contaminating signals with up to four carriers are considered here. Our algorithm proposed here is implemented with both explicit and semi-implicit discretization schemes. Experiments presented here demonstrate promising results indicating a better performance for our nonlinear noise removal method in comparison with the state-of- art in the literature.

1. Introduction

The significance of the orthogonal frequency-division multiplexing (OFDM) over single-carrier modulation techniques is that OFDM can cope with severe channel conditions such as high frequency attenuation in a long wire, narrowband interference and frequency selective fading because of multipath. In the current state-of-the art, the effect of additive noise such as white Gaussian noise (AWGN) and correlated noise is considered to evaluate some characteristics of the system such as its bit error rate (BER) performance (see e.g. [1] and [2]). No concrete work in the literature exists for noise removal from noisy signals with multiple carriers by preserving some key features of the original noiseless signals such as carrier signals and discontinuities separating the consecutive bits. In this paper, we present a nonlinear filter based on the notion of anisotropic diffusion to remove noise from noisy signals and preserve key features of the original noiseless signals such as carrier signals and discontinuities. The notion of anisotropic diffusion as a nonlinear filter to remove the noise and preserve discontinuities in images is introduced by Perona and Malik in [3]. Such a nonlinear filter is generalized for 3D volumetric MRI images in [4] and 2D color images in [5]. A robust algorithm to estimate a piecewise smooth image from the noisy image is developed by Black et al. in [6]. Tschumperie [7] proposes a fast anisotropic smoothing algorithm based on curvature-preserving partial differential equations (PDE) for the noise removal of multi-valued images. A nonlinear band pass filtering algorithm based on anisotropic diffusion for Binary Phase-Shift Keying (BPSK) signals is proposed by Mahmoodi [8]. This algorithm however can be applied to only complex signals and therefore requires calculating the imaginary part of a real valued signal before the filtering process starts. A nonlinear band pass filtering algorithm based on anisotropic diffusion for real valued signals is therefore proposed in [9] to avoid the requirement for the construction of the imaginary part of a real valued signal. Our contributions in this paper are as follows:

i) The anisotropic diffusion based filtering method for band pass signals is extended for carriers with multiple frequencies. This extension enables us to perform the noise removal for OFDM signals by preserving discontinuities and all carrier signals.
ii) The algorithm proposed here is also implemented by employing a semi-implicit discretization scheme in contrast with the previous works in which only explicit discretization technique is used.
iii) Correlated and white Gaussian noise is considered to demonstrate the efficiency of our proposed filtering method.

There are four major differences between the work presented in this paper and the previous works in [8] and [9]:

a) The noise removal methods in [8] and [9] are for signals with a single carrier frequency; however the nonlinear filter presented

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here is extended for signals with multiple carrier frequencies. As demonstrated in this paper, such an extension is not trivial. A numerical experiment is also presented in Section 4 to demonstrate that the noise removal technique for single frequency signals [8] fails for signals with multiple carrier frequencies.

b) The nonlinear filters in [8] and [9] are numerically implemented by using an explicit discretization scheme; however our system in this paper is implemented by employing a semi-implicit discretization technique to increase the numerical stability of our algorithm here in comparison with the algorithm presented in [8] and [9].

c) The noise removal method discussed in [9] is for real valued signals (with a single carrier frequency) and is a special case of the noise removal algorithm for signals with the double carrier frequencies discussed here. Thus the PDE associated with the noise removal presented in [9] possesses real valued coefficients. However PDEs presented in this paper is more general and have complex valued coefficients.

d) The theoretical foundation for double frequency noise removal methods for signals free from discontinuities (linear case) is also established in this paper. As a result, the propagator for a PDE related to a noise removal system for signals with double carrier frequencies is analytically derived here.

The structure of the rest of the paper is as follows. The theory is outlined in Section 2. Section 3 deals with implementation issues. The results are presented in Section 4 and finally conclusions are drawn in Section 5.

2. Theory

The anisotropic diffusion equation, used as a nonlinear noise removal method for low pass signals by Perona and Malik [3] is written as.

$$\left(\frac{\partial}{\partial t}-\left(\frac{\partial}{\partial x}\right)K\left(\frac{\partial}{\partial x}\right)\right)u=0 \tag{1}$$

with initial and boundary conditions:

$$u(x,0)=y(x) \tag{2-a}$$

$$u(0,t)=y(0) \tag{2-b}$$

$$u(L,t)=y(L) \tag{2-c}$$

where $u:\left[0\ \ L\right]\times R^{+}\to R$, $y:\left[0\ \ L\right]\to R$, ($L$ is the length of signal) and $K\geq 0$ are the smoothed low pass signal at the iteration (virtual time) t, the original noisy signal and a function of x respectively. The aforementioned partial differential equation is extended by Mahmoodi [8] for the noise removal of band pass signals with a single carrier with a constant frequency ω_1 as follows:

$$\left(\frac{\partial}{\partial t}-\left(\frac{\partial}{\partial x}-j\omega_1\right)K_1\left(\frac{\partial}{\partial x}-j\omega_1\right)\right)u=0 \tag{3}$$

with initial and boundary conditions:

$$u(x,0)=y(x) \tag{4-a}$$

$$u(0,t)=y(0) \tag{4-b}$$

$$u(L,t)=y(L) \tag{4-c}$$

where $u:\left[0\ \ L\right]\times R^{+}\to R$, $y:\left[0\ \ L\right]\to R$ and $K_1\geq 0$ are the smoothed signal, the noisy signal, and a function of x respectively and also $j=\sqrt{-1}$. Boundary conditions (4-b) and (4-c) are reasonable, because we have no information about the signal at locations $x<0$ or $x>L$. Therefore the values of the smoothed signal at $x=0$ and $x=L$ should remain unchanged. The best guess for the fixed values in these locations is the values of the noisy signal at $x=0$ and $x=L$. Anisotropic diffusion Eq. (3) with initial and boundary conditions (4) provides a method to smooth band pass signals with a single carrier with frequency ω_1.

2.1. Noise removal for signals with two carrier frequencies

In this subsection, we propose a higher order anisotropic diffusion equation for the noise removal of noisy signals with two carrier frequencies. Let us assume that the anisotropic diffusion associated with noisy signals with carrier frequency ω_1 is given by Eqs. (3) and (4) and the anisotropic diffusion equation associated with band bass signals whose carrier frequency is ω_2, can be written as:

$$\left(\frac{\partial}{\partial t}-\left(\frac{\partial}{\partial x}-j\omega_2\right)K_2\left(\frac{\partial}{\partial x}-j\omega_2\right)\right)u=0 \tag{5}$$

with initial and boundary conditions:

$$u(x,0)=y(x) \tag{6-a}$$

$$u(0,t)=y(0) \tag{6-b}$$

$$u(L,t)=y(L) \tag{6-c}$$

To smooth a band pass signal with two carrier frequencies ω_1 and ω_2 such as an OFDM signal, (Eqs. (3) and 5) should be combined. It is noted that any linear combination (i.e. weighted summation/subtraction) of these equations would not be able to smooth a band pass signal with two carrier frequencies. In this paper, we therefore propose the following equation to smooth a band pass signal with carrier frequencies ω_1 and ω_2:

$$\left\{\frac{\partial}{\partial t}-\left(\frac{\partial}{\partial x}-j\omega_1\right)K_1\left(\frac{\partial}{\partial x}-j\omega_1\right)\right\}\left\{\frac{\partial}{\partial t}-\left(\frac{\partial}{\partial x}-j\omega_2\right)K_2\left(\frac{\partial}{\partial x}-j\omega_2\right)\right\}u=0 \tag{7}$$

with initial conditions:

$$u(x,0)=y(x) \tag{8-a}$$

$$\frac{\partial u(x,0)}{\partial t}=0 \tag{8-b}$$

and boundary conditions:

$$u(0,t)=y(0) \tag{9-a}$$

$$u(L,t)=y(L) \tag{9-b}$$

The rationale behind the idea proposed here as Eq. (7) for signals with double carrier frequencies is explained in Appendix A.

PDE (7) can also be written as:

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial t}\left(\left(\frac{\partial}{\partial x}-j\omega_1\right)K_1\left(\frac{\partial}{\partial x}-j\omega_1\right)+\left(\frac{\partial}{\partial x}-j\omega_2\right)K_2\left(\frac{\partial}{\partial x}-j\omega_2\right)\right)u-\left(\frac{\partial}{\partial x}-j\omega_1\right)K_1\left(\frac{\partial}{\partial x}-j\omega_1\right)\left(\frac{\partial}{\partial x}-j\omega_2\right)K_2\left(\frac{\partial}{\partial x}-j\omega_2\right)u \tag{10}$$

Theorem 1. *The propagator of PDE (10) is given by Eq. (11) for a*