Signal Processing 127 (2016) 151–155

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Short communication

# High precision technique for PPS estimation in impulsive noise environment

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ARTICLE INFO

*Article history:*
Received 26 August 2015
Received in revised form
9 January 2016
Accepted 1 March 2016
Available online 9 March 2016

*Keywords:*
Impulsive noise
Polynomial phase signals
ML estimator
Robust TF representations

ABSTRACT

A technique for high precision parametric estimation of high-order polynomial phase signals (PPS) is introduced that can be used in impulsive noise environments. It is inspired by the recently proposed quasi maximum likelihood (QML) estimator with several modifications in coarse and fine estimation stages as well as in the optimization criterion. The developed technique achieves improvement in the mean squared error (MSE) for more than 20 dB with respect to the standard QML for high-order PPSs of short length and up to 20% of impulses.



## 1. Introduction

It has been assumed for a long time that a significant loss of accuracy cannot be avoided in the polynomial phase signals (PPS) estimation with the PPS order increase [1–7]. However, this loss of accuracy can be significantly reduced by employing the quasi-maximum likelihood (QML) estimator for the Gaussian noise environment [2,9–11]. The QML estimator provides excellent results with a low signal-to-noise ratio (SNR) threshold (more than 10 dB better than the current state-of-the-art techniques) and with the mean squared error (MSE) on the Cramer–Rao lower bound (CRLB) above the SNR threshold. However, an especially important case of signals corrupted by impulsive noise has not been addressed. In this paper we describe a technique that suppresses the impulsive noise in the received signal and provides the accurate PPS estimates. The crucial components in the proposed technique are the robust short time Fourier transform (STFT) able to suppress the impulsive noise, and a nonlinear filter used in the refinement stage [12–19].

The paper is organized as follows. In Section 2, the signal model is presented along with the QML estimation algorithm. In Section 3, crucial steps in the algorithm making it robust to impulsive noise are explained. A numerical example in Section 4 affirms the accuracy of the proposed technique.

## 2. Signal model and PPS estimator

### 2.1. Signal model

The following PPS model [1–11]

$$x(t) = A\exp(j\phi(t)) + \nu(t) = A\exp\left(ja_0 + \sum_{m=1}^{M} a_m t^m/m\right) + \nu(t), \tag{1}$$

is considered, where $A$ is the signal amplitude, $\phi(t)$ is the signal phase, while $\nu(t)$ is an additive noise. The instantaneous frequency (IF) is defined as the first derivative of the signal phase $\omega(t) = \phi'(t)$. The signal is sampled with the sampling rate $\Delta t$, $x(n) = x(n\Delta t)$. The goal is to estimate parameters of the PPS $\{a_0, a_1, a_2, \ldots, a_M\}$ from noisy observations, where $M$ is the order of the phase polynomial. This

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http://dx.doi.org/10.1016/j.sigpro.2016.03.002
0165-1684/

is a problem of significant practical importance in various fields, especially in radar systems.

The Gaussian noise environment is commonly assumed. Here, we consider practically important impulsive noise environments treated lightly in the literature [20].

### 2.2. QML algorithm

The QML estimator can be described by the following algorithm.

*Coarse stage*:

1. Calculate the time-frequency (TF) representation $TF_h(t,\omega)$ of the considered signal. Parameter $h$ is the window width. The window width influences the TF representation-based IF estimator performance [21,22]. Narrower windows exhibit smaller bias but are still sensitive to noise, whereas for wider windows the IF estimator has the emphatic bias with significantly reduced noise influence [21,22]. The STFT is used as the TF representation [2]:

$$\begin{aligned} TF_h(t,\omega) = STFT_h(t,\omega) &= \frac{1}{(h\Delta t)} \sum_{n=-h/2\Delta t}^{h/2\Delta t - 1} x(t+n(\Delta t)) \\ &\quad \times \exp(-j\omega n(\Delta t)) \\ &= \mathrm{mean}\{x(t+n(\Delta t)) \\ &\quad \times \exp(-j\omega n(\Delta t)) | n \in [-h/2\Delta t, h/2\Delta t)\}. \end{aligned} \tag{2}$$

2. The IF is estimated using the position of the TF maxima,

$$\hat{\omega}_h(t) = \arg\max_{\omega} |TF_h(t,\omega)|. \tag{3}$$

3. Coarse estimation of signal parameters is performed using classical polynomial regression of $\hat{\omega}_h(t)$. This estimate is denoted as $\{\hat{a}^h_m | m \in [1,M]\}$.

*Fine stage*:

4. Dechirping, filtering, and phase unwrapping [11]

$$v(t) = \mathrm{unwrap}\left(\mathrm{phase}\left(F\left\{x(t)\exp\left(-j\sum_{m=1}^{M} \hat{a}^h_m t^m/m\right)\right\}\right)\right) \tag{4}$$

where $F\{\}$ is a low pass filtering operator. Dechirping $x(t)$ by $\exp\left(-j\sum_{m=1}^{M} \hat{a}^h_m t^m/m\right)$ yields a lowpass signal that can be filtered without distortion. Furthermore, it is suitable for phase unwrapping. Signal $v(t)$ (unwrapped phase of the dechirped signal) is a polynomial of the same order as the polynomial in the signal phase. The theoretical properties of the phase unwrapping estimators under weak assumptions on the noise distribution are considered in [23], while identifiability and aliasing in the PPS are considered in [24]. A simple moving average filter is used as a low pass operator $F\{\}$ in [2]. Signal $v(t)$ represents the difference between true and estimated signal phases in the coarse stage.

5. Perform the polynomial regression of $v(t)$ to estimate its coefficients. The fine estimate of the phase parameters is obtained through summation of these estimates and the results from the coarse stage. This is referred to as the O'Shea refinement strategy [11]. The fine estimate is denoted as $\{\tilde{a}^h_m | m \in [0,M]\}$.

*Final estimate*:

6. Calculate the optimization function

$$J(h) = \left| \sum_n x(n\Delta t)\exp\left(-j\sum_{m=1}^{M} \tilde{a}^h_m (n\Delta t)^m\right)\right| \tag{5}$$

for the obtained estimates $\{\tilde{a}^h_m | m \in [0,M]\}$. Maximization of $J(h)$ gives the final estimate

$$\begin{aligned} \hat{h} &= \arg\max_h J(h), \\ \hat{a}_m &= \tilde{a}^{\hat{h}}_m, \quad m \in [0,M]. \end{aligned} \tag{6}$$

## 3. Proposed estimator for impulse noise environment

We have presented the general form of the QML algorithm with three functions: TF representation $TF_h(t,\omega)$, filtering scheme $F\{\}$, and optimization function $J(h)$. Now we propose the alternative forms of these functions, having as an input PPS estimates robust to the impulsive noise influence.

*Robust STFT*: It is well known that the standard TF representations are inaccurate for impulsive noise environments. Therefore, the robust TF representations are proposed [13]. Here, the robust STFT in the marginal-median form [13,19]:

$$\begin{aligned} TF_h(t,\omega) = rSTFT_h(t,\omega) = \mathrm{median}\{&\mathrm{Re}\{x(t+n(\Delta t)) \\ &\times \exp(-j\omega n(\Delta t))\} | n \in [-h/2\Delta t, h/2\Delta t)\} \\ &+ j\,\mathrm{median}\{\mathrm{Im}\{x(t+n(\Delta t))\exp(-j\omega n(\Delta t))\} | n \in \\ &\times [-h/2\Delta t, h/2\Delta t)\}, \end{aligned} \tag{7}$$

is applied. Alternative robust STFTs can also be used [13,19].

The robust STFT is used as the IF estimator in (3). Performance of the IF estimator based on the robust STFT is considered in detail in [25]. The robust STFT is a biased IF estimator for PPSs of order higher than $M=2$. The bias satisfies $E\{\hat{\omega}_h(t) - \omega(t)\} \approx ah^2\omega^{(2)}(t)$, where $\omega^{(2)}(t)$ is the second IF derivative, while $a$ is a constant (for details see [25]). Note that the bias is the same for both standard and robust STFTs. It increases with the increase of the window width. The variance of the IF estimator can be written as $E\{(\hat{\omega}_h(t) - \omega(t))\} \approx b_{TF,\nu}/h^3$ where $b_{TF,\nu}$ is a constant depending on the noise environment and on the applied STFT form [25]. The variance decreases with the increase of the window width. Obviously, there is an optimal $h$ producing minimal MSE as a bias to variance trade-off. This value varies with a noise environment and the STFT version used. For an impulsive noise environment, $b_{TF,\nu}$ for the standard STFT rapidly increases. For the considered impulsive environment, $b_{TF,\nu}$ increases moderately for the robust STFT with respect to the Gaussian noise environment. This means that the optimal window width gradually increases for more emphatic noise.

*Filtering scheme*: The second issue is the lowpass filtering operator $F\{\}$ in the fine estimation stage. There is an important difference here with respect to the coarse stage. Namely, in the coarse stage the signal of interest is a high frequency one, so the standard lowpass robust filters cannot be used. This is the reason for using the robust TF representations. However, after dechirping (step 4 of the algorithm) signal spectral content is moved to low frequencies and it can be filtered with the standard robust filters. Here, the marginal-median filter of the