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Eigenvalue Analysis of sound propagation characteristics in a circular duct lined with poroelastic foams

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ABSTRACT

An effective method for eigenvalue analysis of a circular duct lined with poroelastic foams is presented using axisymmetric finite element models based on Biot's theory and Helmholtz equation. Complex dispersion relations in a cylindrical foam-lined duct are successfully identified using an iterative Prony series method. It is shown that the numerical results obtained by the proposed method agree well with those obtained by measurements and direct forced response simulations. The influences of thickness and boundary conditions of the poroelastic foam on the sound propagation characteristics in a duct are also investigated. Furthermore, the damping effect due to viscosity of the foam on dispersion curves is discussed at a theoretical level.

1. Introduction

The work described here relates to sound propagation in a duct lined with porous materials. Generally, sound propagation in a lined duct has been studied by taking into consideration in-duct acoustical properties such as sound attenuation. Acoustical models with simple geometries have been established under the assumption of locally- or bulk-reacting linings. Morse [1] proposed a simplified model for sound propagation in a lined duct. He derived a characteristic equation that predicts sound attenuation by assuming the surface of foam linings as locally reacting, and expressed the absorptive effect of the liner on a sound field in airway in terms of normal incidence specific acoustic impedance of the lining. Later, Scott [2] found that Morse's model is not applicable when the lining material is loosely packed or when the width of airway in the duct relative to the wavelength is small. By assuming the liners as being extensively reacting, he derived a characteristic equation of his model more accurate than that of Morse's model, which governed sound fields in both the liner and the airway. However, both models have limitations in their applications to cases involving lined ducts with arbitrary shaped cross-sections and number of propagating wave types. Kang and Jung [3] presented sound attenuation and phase speed in circular ducts lined with poroelastic noise control foam which allows three types of wave propagations.

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Application of Finite Element Method to the lined ducts made it possible to study ducts of arbitrary shapes of cross-sections. Accordingly, FEM has been adopted to solve problems related to porous materials. Astley and Cummings [4] presented a finite element formulation for the prediction of attenuation and phase speed in a duct with extensively reacting porous liner. The finite element formulation by using axisymmetric foam have been developed and validated in Refs. [5,6].

Besides the forced response analysis, acoustical properties such as sound attenuation and phase velocity in lined duct can be obtained from eigenvalue analysis. Various approaches to eigenvalue analysis have been proposed by many researchers. Ko [7] solved the eigenvalue problems in circular flow ducts by expressing the radial function of the disturbance velocity potential in Taylor series in his governing equation, and suggested that the technique could be extended to the case with no flow. Then, he presented the sound attenuation obtained from calculated eigenvalues in different velocity profiles. On the other hand, Astley and Eversman [8] considered finite element method to the eigenvalue problem for a lined duct and argued that spurious modes could be eliminated by using a condensation technique and continuous slope shape functions. It has been found that generating eigenvalue solutions based on finite element discretization are the most efficient approaches to problems involving sound propagation in ducts.

A number of studies have been concerned with guided waves in porous materials. Boeckx and Allard [9,10] investigated phase velocities of guided waves in porous layers and proposed a new experimental measurement method for phase velocity. In addition, Wisse et al. [11,12] extended the classical theory of wave propagation in elastic cylinders to poroelastic mandrel modes. They also indicated that all waves in poroelastic mandrels become damped due to viscosity effects and bulk wave modes occur owing to the existence of the Biot slow wave.

In this paper, we considered the characteristics of wave propagation in a coupled system of poroelastic foam and airway using eigenvalue analysis. Based on Biot's theory and Helmholtz equation, complex dispersion relations in a cylindrical foam-lined duct have been identified and the characteristics of fundamental quantities such as, attenuation and phase speed have been illustrated analytically and directly unlike measurements and forced response analysis. Also this paper theoretically accounts for the effects of mode contamination at high frequencies, which are commonly observed in measurements and forced response analysis. The results of the eigenvalue analysis are validated by comparison with dispersion curves in frequency-wave number domain, as well as sound attenuation and phase speed of the measurements and forced response simulation which are calculated by Prony Series from spatial data of airway.

2. Eigenvalue analysis

2.1. Eigenvalue formulation for the liner

Axisymmetric foam finite element model used in this paper was developed from the elastic porous material theory of Shiau and Bolton [13,14] which itself is based on the theory of Biot [15]. Combining stress–strain relations and dynamic equations that describe motions of both solid and fluid phases in axisymmetric cylindrical coordinates leads to the following two differential equations relating stresses and displacements:

$$N\nabla^2\mathbf{u}+\nabla[(A+N)e_s+Q\varepsilon]=-\omega^2(\rho_{11}\mathbf{u}+\rho_{12}\mathbf{U})+j\omega b(\mathbf{u}-\mathbf{U}) \tag{1}$$

$$\nabla[Qe_s+R\varepsilon]=-\omega^2(\rho_{12}\mathbf{u}+\rho_{22}\mathbf{U})-j\omega b(\mathbf{u}-\mathbf{U}) \tag{2}$$

where e_s and ε are, respectively, the solid and fluid volumetric strains, $\mathbf{u}$ and $\mathbf{U}$ are the displacement vector of solid phase and fluid phase, respectively. N is the shear modulus, and A the Lamé constant. The coefficient R is the stiffness of the interstitial fluid, and the coefficient Q the coupling between volume changes of solid and fluid phases of the material. Mass coefficients ρ_{11}, ρ_{12}, and ρ_{22} account for the inertias of solid and fluid phases, and the effects of momentum transfer between the two phases, as well as the viscous coupling forces proportional to the relative velocity of the two phases. The solid and fluid volumetric strains can be written in axisymmetric form as follows:

$$e_s=\nabla\mathbf{u}=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z} \tag{3}$$

$$\varepsilon=\nabla\mathbf{U}=\frac{\partial U_r}{\partial r}+\frac{U_r}{r}+\frac{\partial U_z}{\partial z} \tag{4}$$

where u_r, u_z, U_r, U_z are the radial and axial components of the solid and fluid phase displacements, respectively.

To develop a weak form of differential Eqs. (1) and (2), $\Omega^e$is assumed to be an arbitrary and typical element, then the finite element model based on two differential equations is developed over Ω^e. By multiplying Eqs. (1) and (2) by the appropriate weighting functions, $\psi_i(i=1,2,3,4)$, which themselves are assumed to be once differentiable with respect to r and z, the derived equations can be integrated over the element domain Ω^e, so that the weighting functions and displacement variables are equally differentiated. Thus, one obtains

$$0=2\pi\int_{\Omega^e}\left[r\frac{\partial\psi_1}{\partial r}\left(2N\frac{\partial u_r}{\partial r}+A\nabla\mathbf{u}+Q\nabla\mathbf{U}\right)+\psi_1\left(2N\frac{u_r}{r}+A\nabla\mathbf{u}+Q\nabla\mathbf{U}\right)\right.$$
$$\left.+r\frac{\partial\psi_1}{\partial z}N\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)-\omega^2\rho_{11}^*r\psi_1u_r-\omega^2\rho_{12}^*r\psi_1U_r\right]dr\,dz$$