Signal Processing 117 (2015) 138–150

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

# Adaptive active contour model driven by fractional order fitting energy

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ARTICLE INFO

*Article history:*
Received 20 October 2014
Received in revised form
8 April 2015
Accepted 12 May 2015
Available online 21 May 2015

*Keywords:*
Image segmentation
Fractional order differentiation
Level set method
Variational method
Active contours

ABSTRACT

In this paper, a new adaptive active contour model is proposed for image segmentation, which is built based on fractional order differentiation, level set method and curve evolution. The energy functional for the proposed model consists of three terms: fitting term, regularization tern and penalty term. By incorporating the fractional order fitting term, the novel fitting term can describe the original image more accurately, and be robustness to noise. In order to ensure stable evolution of the level set function, a penalty tern is added into the proposed model. The results evolution of the level set function is the gradient flow that minimizes the overall energy functional. Experimental results for both synthetic and real image show desirable performance of our method.



## 1. Introduction

Image segmentation is a functional and key problem in image processing and computer vision, especially in application in biomedical image processing. Its goal is to divide a given image into its constituent part that is more meaningful and easier to analyze. Up to now, a broad variety of models have been proposed to solve the image segmentation problem, in which geometric active contour model using the level set method is a class of efficient techniques.

Active contour model, proposed by Kass et al. [1], has been proved to be an efficient framework for image segmentation. It segments objects images using dynamic curves based on an energy-minimizing model. However, this method suffers from the sensitivity to initial conditions and the difficulties associated with topological changes like the merging and splitting of the evolving curve. After the active contour model is proposed, many researchers have done great efforts to improve the performance of it. The most important and successful one is implicit active contour model, i.e., active contour model in a level set formulation, which is originally introduced by Osher and Sethian [2]. The fundamental ideal is to present contours as the zero level set of a function in higher dimension (called level set function), and then to evolve the level set function according to an evolution partial differential equation (PDE). The main advantage of this method is to handle the topological changes automatically. This is generally impossible for the traditional parametric active contour model. In the view of mathematics, implicit active contour models can be categorized into two categories: one is pure PDE method [2–6] whose evolution equation is directly constructed; another is the variational level set method [7–11] whose evolution equation is derived from the minimization problem for the energy functional defined on the level set function. Compared with the pure PDE method, the varialtional level set method is more convenient and natural for incorporating prior information, such as shape and intensity distribution, into a principled energy functional [4]. Thus, we focus on the variational level set method in this paper.

The variational level set methods have been well established and widely used in many application of image processing. In the field of image segmentation, we solve

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http://dx.doi.org/10.1016/j.sigpro.2015.05.009
0165-1684/

this problem by deriving an energy functional from some a priori mathematical model and minimizing this energy functional over all possible partitions. The popular piecewise constant (PC) model [12] is a typical variational level set method, which aims to minimize the Mumford – Shah functional [13]. They incorporate the global image statistics inside and outside the evolving curve into their energy functional as a prior constraint. However, the PC model usually fails to segment the images with intensity inhomogeneity since it assumes that the intensities in each region always maintain constant. To improve the performance of the PC model, many efficient models have been proposed in related literatures. Wang et al. formulate a local PC model using local statistical function and extend this model for texture segmentation by an extended structure tensor [14]. Li et al. proposed an implicit active contours model based on local binary fitting energy, which used the local information as constraint, and well work on the image with intensity inhomogeneities [15,16]. Zhang et al. proposed a novel region-based active contour model for image segmentation, which combined the merits of the traditional GAC and PC models [17]. He et al. propose an improved region-scalable fitting model based on the “mollifying” kernel and local entropy [18]. Weng et al. develop the PS model by using $L^p$ norm instead of $L^2$ norm to define the fitting energy, which could fit the image gradient information adaptively [19]. Zhang et al. exploited local image region statistics to present a level set method for segmenting images with intensity inhomogeneity [20]. These works show that a reasonable fitting energy is very important for their models. In this paper, we use the property of fractional order differentiation to design the fitting energy, and enable it to fit the original image accurately.

Up to now, fractional order differentiations/equations have been widely applied in many areas, especially in the filed of image processing [21–23]. Mathieu et al. proposed an edge detector based on fractional differentiation [24]. Nakib et al. gave a new geometric interpretation of the two dimensional fractional differentiation and applied it into threshold segmentation [25]. Ren et al. present novel a image up-sampling algorithm based on fractional-order bidirectional diffusion [26]. Unlike the integer order differentiation, the fractional order differentiation is of the non-local property. It has many methods that can define the fractional order differentiation such as Riemann – Liouville fractional order differentiation, Grünwald – Letnikov fractional order differentiation and frequency-domain fractional order differentiation. As we known, fractional order differentiation is a nonlinear attenuator. The enhancement and preservation of low frequency information by fractional order differentiation is superior to that by first order one [27]. This is very significant in image processing.

In variational level set methods, some regularization should be imposed on the level set cure to diminish the influence of noise and smooth the level set function. In the spirit of the Mumford–Shah (MS) functional, length regularization is a popular choice of the geometric constraint to penalize the length of contours. This regularization is less robustness to noise. In the later research, smoother regularizations have been used in image segmentation, such as regularizations based on $L^2$ norm or $L^\infty$norm[28]. Although these regularizations diminish the influence of noise, they cause the evolution curve to pass through weak edges. Wu and Tai present geodesic curvature flow over general surfaces using the level set function via a weighted curve [29]. Li et al. use the variable exponent $p$-Dirichlet integral to form a novel weighted regularization to diminish the influence of noise [30]. Based on these methods, we adopt a weighted regularization based on a novel edge indicator function in this paper.

In this paper, we present a new adaptive active contour model based on fractional order differentiation. The energy functional for the proposed model consists of three terms: fitting term, regularization tern and penalty term. We combine the fractional order fitting term with the global fitting term to form a novel fitting term. By incorporating the fractional order fitting term, the novel fitting term can describe the original image more accurately, and be robustness to noise. In order to ensure stable evolution of the level set function, a penalty tern is added into the proposed model. In addition, an adaptive function is incorporated into the traditional length regularization to smooth the level set function. The results evolution of the level set function is the gradient flow that minimizes the overall energy functional. Experimental results for both synthetic and real image show desirable performance of our method.

The remainder of the paper is organized as follows. In Section 2, we present a brief introduction of Mumford – Shah model, PC model and fractional order differentiation. The proposed model and its analysis are introduced in Section 3. We present the numerical scheme and experimental results in Section 4. The conclusion is presented in Section 5.

## 2. Background

In this section, we briefly review Mumford – Shah model (Section 2.1), Piecewise Constant model (Section 2.2) and fractional order differentiation (Section 2.3).

### 2.1. Mumford – Shah model

Mumford – Shah model is an energy-based image segmentation method introduced by Mumford and Shah. Let $\Omega \subset \mathbb{R}^2$be the image domain, $u_0 : \Omega \to \mathbb{R}$ be a given gray image. The basic idea of Mumford-Shah model is to seek a smooth curve $C$that divides image $u_0$ into several non-overlapping regions, and a piecewise smooth approximate $u$ of $u_0$. Their model can be formulated as

$$E^{MS}(u, C) = \int_{\Omega} |u - u_0|^2 dxdy + \mu \int_{\Omega \setminus C} |\nabla u|^2 dxdy + \nu |C|, \tag{1}$$

where $\mu$ and $\nu$ are positive constants, $|C|$is the length of curve$C$. Thus, to perform image segmentation task is to minimize the functional (1) over $u$ and $C$. Because of the unknown curve $C$ of lower dimension and the nonconvexity of the functional (1), it is difficult to minimize the Mumford – Shah model. In practice, some alternative methods have been done to simply or modify the functional (1), including piecewise constant model introduced as follows.