Signal Processing 128 (2016) 382–388

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Short communication

Persymmetric detectors of distributed targets in partially homogeneous disturbance

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ARTICLE INFO

Article history:
Received 29 January 2016
Received in revised form
21 April 2016
Accepted 6 May 2016
Available online 9 May 2016

Keywords:
Adaptive radar detection
Rao test
Wald test
Distributed target
Persymmetry

ABSTRACT

This paper deals with the problem of detecting distributed target in Gaussian disturbance with unknown but persymmetric structured covariance matrix. The partially-homogeneous environment is considered and two receivers based on the Rao test and the Wald test design criteria are derived at the design stage. The performance assessment conducted by resorting to both simulated data and real data, also in comparison to the previously proposed detectors, has confirmed the effectiveness of the newly proposed detectors.

1. Introduction

The size of a target in terms of range cells depends on both the physical size of the target and the ratio between the latter and sensor resolution. A target is considered extended whenever it produces backscattering centers in adjacent range bins [1]. A high-resolution radar (HRR) can resolve a target into a number of scattering centers appearing into different range cells [2]. Moreover, it is also well known that the point-like target model may fail in many scenarios wherein a low/medium resolution radar is employed: examples of these situations are the detection of large ships with coastal radars and that of a cluster of point targets flying at the same velocity in close spatial proximity to one another. Thus, the detection of distributed targets has received an extensive attention in recent years [3–11].

Adaptive detectors relying on the generalized likelihood ratio test (GLRT) [6,7] have been proposed over the years assuming homogeneous scenario, where the cell under test and the training data share the same spectral properties of the interference. However, the homogeneous model is invalid in realistic clutter environments and the partially homogeneous model has been proposed. This model assumes that the secondary data share the same covariance structure as the primary data but different power level [9]. Later, the one-step GLRT-based and two-step GLRT-based detectors [10], the Rao and Wald detectors [11] have been devised for partially homogeneous scenario.

Conventional receivers suffer matched detection performance degradation in realistic scenario due to the fact that the number of secondary data is always limited. The structural information about the disturbance covariance matrix represents a viable means to face with the problem. It has been shown in [12] that if the elements of a line array are symmetrically spaced with respect to the phase center, the covariance matrix of the disturbance is persymmetric. Specifically, it has a doubly symmetric form, namely, it is Hermitian about its principal diagonal and persymmetric about its cross diagonal [13–15]. Such a situation is frequently met in radar systems [14]. Several works concerning the persymmetric property can be found in [12–23]. For the problem of detecting point-like targets, a persymmetric GLRT detector [15], persymmetric Rao and Wald detectors [16] and the persymmetric adaptive normalized matched filter [17] have been proposed. Other examples of persymmetric receivers can be found in [13] and [18–20]. Recently, persymmetry is used in conjunction with invariance [21] and MIMO radar [22]. To the best of authors' knowledge the design of persymmetric detectors for distributed targets has received less consideration. In [23], a persymmetric one-step GLRT (P1S-GLRT-PH) and a persymmetric two-step GLRT (P2S-GLRT-PH) for detecting distributed targets have been proposed. As alternative strategies with respect to the GLRT, Rao and Wald tests might be more robust than the GLRT under mismatched

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http://dx.doi.org/10.1016/j.sigpro.2016.05.010
0165-1684/

conditions (occurring in real operating situations) and might require a smaller computational complexity than the GLRT for their implementation [24]. Therefore, it is of interest to investigate Rao and Wald tests for the same problem.

Motivated by the above mentioned works, we focus on the problem of detecting distributed target embedded in partially-homogeneous Gaussian disturbance with unknown covariance matrix. Precisely, we assume a persymmetric structured covariance matrix of the disturbance and apply Rao and Wald tests to derive two new receivers referred to as the Per-Rao and the Per-Wald. Then the matched detection performance of the new designed Per-Rao and Per-Wald are assessed in comparison with their unstructured counterparts which do not exploit the persymmetry property of the disturbance covariance matrix. Experimental results show that the Per-Rao and the Per-Wald can improve the matched detection performance with a small set of secondary data. Additionally, both matched and mismatched detection performance are analyzed in comparison with the P1S-GLRT-PH and P2S-GLRT-PH detectors. Simulation results show that the Per-Rao exhibits better rejection capabilities of mismatched signals than the P1S-GLRT-PH and the P2S-GLRT-PH with a certain matched detection loss and the Per-Wald exhibits worse rejection capabilities of mismatched signals but nearly the same matched detection performance as the P2S-GLRT-PH. Finally, real sea clutter data are used to test the new detectors further. The proposed Per-Rao and Per-Wald detectors and their performance analysis represent the novel contributions of this paper.

2. Problem formulation

We assume that data are collected from N sensors [25] and the target occupies L range cells. The primary data and the secondary data which do not contain any useful target signal are denoted by $\boldsymbol{z}_t \in C^{N\times 1}$, $t = 1, \ldots, L$ and $\boldsymbol{z}_t \in C^{N\times 1}$, $t = L+1, \ldots, L+K$, respectively. Here, t denotes the range cell. We want to decide whether $\boldsymbol{z}_t \in C^{N\times 1}$, $t = 1, \ldots, L$ contain target signals or not. The detection problem at hand can be formulated as the following binary hypotheses test

$$\begin{cases} H_0\colon \boldsymbol{z}_t = \boldsymbol{n}_t, & t = 1, \ldots, L+K, \\ H_1\colon \begin{cases} \boldsymbol{z}_t = \alpha_t \boldsymbol{p} + \boldsymbol{n}_t, & t = 1, \ldots, L, \\ \boldsymbol{z}_t = \boldsymbol{n}_t, & t = L+1, \ldots L+K, \end{cases} \end{cases} \tag{1}$$

where $\boldsymbol{p} \in C^{N\times 1}$ is the nominal steering vector; α_ts, $t = 1, \ldots, L$ are unknown deterministic parameters accounting for the target reflectivity and the channel effects; $\boldsymbol{n}_t \in C^{N\times 1}$, $t = 1, \ldots, L+K$ are circularly symmetric, independent complex Gaussian vectors with zero mean and covariance matrix

$$\begin{cases} E\left[\boldsymbol{n}_t \boldsymbol{n}_t^H\right] = \boldsymbol{M}, & t = 1, \ldots, L, \\ E\left[\boldsymbol{n}_t \boldsymbol{n}_t^H\right] = \gamma \boldsymbol{M}, & t = L+1, \ldots, L+K, \end{cases} \tag{2}$$

where $E[\cdot]$ denotes statistical expectation operator, $(\cdot)^H$denotes conjugate transpose, and $\gamma > 0$. Note that $\gamma = 1$ represents the homogeneous scenario.

Assume that covariance matrix $\boldsymbol{M}$ has the persymmetric property [15], namely,

$$M = J\boldsymbol{M}^*\boldsymbol{J} \tag{3}$$

where $(\cdot)^*$ denotes complex conjugate, $\boldsymbol{J} \in R^{N\times N}$ is a permutation matrix, i.e., if $i + j = N + 1$, $\boldsymbol{J}(i, j) = 1$, otherwise, $\boldsymbol{J}(i, j) = 0$. Here, we also assume that the steering vector $\boldsymbol{p}$satisfies $\boldsymbol{p} = \boldsymbol{J}\boldsymbol{p}^*$. In this paper, we resort to the Rao and Wald tests to solve the binary hypothesis testing problem in (1).

3. Design of two persymmetric detectors

To solve the problem at hand, we resort to the Rao and Wald tests. We denote by

$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_L]$ a L-dimensional complex row vector; $\boldsymbol{Z}_L = [\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_L]$ the primary data;

$\boldsymbol{\theta} = \left[\boldsymbol{\theta}_r^T, \boldsymbol{\theta}_s^T\right]^T$a $(2L + N^2 + 1)$ -dimensional real column vector;

$\boldsymbol{\theta}_r = [\alpha_{1,R}, \alpha_{1,I}, \ldots, \alpha_{L,R}, \alpha_{L,I}]^T$a $2L$-dimensional real column vector, where $\alpha_{t,R}$ and $\alpha_{t,I}$ denote the real part and imaginary part of α_t, $t = 1, \ldots, L$, $(\cdot)^T$denotes transpose;

$\boldsymbol{\theta}_s = [\gamma, \Xi(\boldsymbol{M})^T]^T$an $(N^2 + 1)$-dimensional real column vector, with $\Xi(\boldsymbol{M})$ the one-to-one operator mapping $\boldsymbol{M}$ to $\boldsymbol{\theta}_s$.

3.1. Persymmetric Rao test

The Rao test to solve the problem can be expressed as the decision rule in the following [26]

$$\left.\frac{\partial \ln f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M})}{\partial \boldsymbol{\theta}_r}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_0}^T \left[\boldsymbol{J}^{-1}\left(\hat{\boldsymbol{\theta}}_0\right)\right]_{\boldsymbol{\theta}_r,\boldsymbol{\theta}_r} \times \left.\frac{\partial \ln f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M})}{\partial \boldsymbol{\theta}_r}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_0} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_R \tag{4}$$

where $\partial/\partial\boldsymbol{\theta}_r$ denotes the gradient with respect to $\boldsymbol{\theta}_r$ (namely, $\partial/\partial\boldsymbol{\theta}_r = \left[\partial/\partial\alpha_{1,R}, \partial/\partial\alpha_{1,I}, \ldots, \partial/\partial\alpha_{L,R}, \partial/\partial\alpha_{L,I}\right]^T$); $\hat{\boldsymbol{\theta}}_0 = \left[\hat{\boldsymbol{\theta}}_{r,0}^T, \hat{\boldsymbol{\theta}}_{s,0}^T\right]^T$ is the maximum likelihood estimate (MLE) of $\boldsymbol{\theta}$ under H_0; $\boldsymbol{J}(\boldsymbol{\theta}) = \boldsymbol{J}(\boldsymbol{\theta}_r, \boldsymbol{\theta}_s)$ denotes the Fisher information matrix and can be partitioned as [26] $\boldsymbol{J}(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{J}_{\theta_r,\theta_r}(\boldsymbol{\theta}) & \boldsymbol{J}_{\theta_r,\theta_s}(\boldsymbol{\theta}) \\ \boldsymbol{J}_{\theta_s,\theta_r}(\boldsymbol{\theta}) & \boldsymbol{J}_{\theta_s,\theta_s}(\boldsymbol{\theta}) \end{bmatrix}$, where $\left[\boldsymbol{J}_{\theta_r,\theta_r}(\boldsymbol{\theta})\right] = -E\left[\frac{\partial^2 \ln f(\boldsymbol{z}_{1:L+K}|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_r \partial\boldsymbol{\theta}_r^T}\right]$, $\left[\boldsymbol{J}_{\theta_r,\theta_s}(\boldsymbol{\theta})\right] = -E\left[\frac{\partial^2 \ln f(\boldsymbol{z}_{1:L+K}|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_r \partial\boldsymbol{\theta}_s^T}\right]$, $\left[\boldsymbol{J}_{\theta_s,\theta_r}(\boldsymbol{\theta})\right] = -E\left[\frac{\partial^2 \ln f(\boldsymbol{z}_{1:L+K}|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_s \partial\boldsymbol{\theta}_r^T}\right]$, $\left[\boldsymbol{J}_{\theta_s,\theta_s}(\boldsymbol{\theta})\right] = -E\left[\frac{\partial^2 \ln f(\boldsymbol{z}_{1:L+K}|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_s \partial\boldsymbol{\theta}_s^T}\right]$. η_R denotes the detection threshold which can be determined by Monte Carlo techniques. $f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M})$ denotes the probability density function (PDF) of the data under H_1, and its expression is $f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M}) = \frac{\gamma^{-NK}}{\pi^{N(L+K)}\det^{L+K}(\boldsymbol{M})} \times \exp\left\{-tr\left[\boldsymbol{M}^{-1}\left((\boldsymbol{Z}_L - \boldsymbol{p}\boldsymbol{\alpha})(\boldsymbol{Z}_L - \boldsymbol{p}\boldsymbol{\alpha})^H + \frac{1}{\gamma}\boldsymbol{S}\right)\right]\right\}$, where $\det(\cdot)$ denotes the determinant operator and $tr(\cdot)$ denotes the trace operator, respectively, $\boldsymbol{S} = \sum_{t=L+1}^{L+K} \boldsymbol{z}_t \boldsymbol{z}_t^H$.

After some calculation, we can obtain

$$\frac{\partial \ln f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M})}{\partial \alpha_{t,R}} = 2\,\mathrm{Re}\left\{\boldsymbol{p}^H \boldsymbol{M}^{-1}(\boldsymbol{z}_t - \alpha_t \boldsymbol{p})\right\} \tag{5}$$

$$\frac{\partial \ln f(\boldsymbol{z}_1, \ldots \boldsymbol{z}_L, \ldots \boldsymbol{z}_{L+K}|\boldsymbol{\theta}, \boldsymbol{M})}{\partial \alpha_{t,I}} = 2\mathrm{Im}\left\{\boldsymbol{p}^H \boldsymbol{M}^{-1}(\boldsymbol{z}_t - \alpha_t \boldsymbol{p})\right\} \tag{6}$$

$$\begin{aligned} \left[\boldsymbol{J}^{-1}(\boldsymbol{\theta})\right]_{\theta_r,\theta_r} &= \left(\boldsymbol{J}_{\theta_r,\theta_r}(\boldsymbol{\theta}) - \boldsymbol{J}_{\theta_r,\theta_s}(\boldsymbol{\theta}) \boldsymbol{J}_{\theta_s,\theta_s}^{-1}(\boldsymbol{\theta}) \boldsymbol{J}_{\theta_s,\theta_r}(\boldsymbol{\theta})\right)^{-1} = \boldsymbol{J}_{\theta_r,\theta_r}^{-1}(\boldsymbol{\theta}) \\ &= \frac{1}{2}\left(\boldsymbol{p}^H \boldsymbol{M}^{-1} \boldsymbol{p}\right)^{-1} \mathbf{I}_{2L\times 2L} \end{aligned} \tag{7}$$

where $\mathbf{I}_{2L\times 2L}$ denotes a $2L \times 2L$ identity matrix.

From (4), the MLE of $\boldsymbol{\theta}$ under H_0 (i.e., $\hat{\boldsymbol{\theta}}_0$) is also required to obtain the Rao test. By exploiting the persymmetric property, the PDF of the data under H_0 can be rewritten as [15]