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Energy optimized orthonormal wavelet filter bank with prescribed sharpness

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A R T I C L E I N F O

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A B S T R A C T

Optimization with respect to some energy measure such as compaction energy is a widely used criterion for designing wavelet filter banks. The filter bank can be adapted to the signal that it is analyzing to achieve good performance. The frequency selectivity property of a traditional low-pass filter is however not ensured using this criterion. Frequency selectivity is important to ensure the effects on aliasing is minimized in the subband and to give a regular equivalent wavelet function. In this work the design of energy optimized filters with a prescribed sharpness in the frequency response is presented. The sharpness, which determines the degree of selectivity, is achieved by the zero-pinning technique on the Bernstein polynomial. The design technique can be cast as a Semidefinite Programming (SDP) problem which can be solved with efficient interior point algorithms.

1. Introduction

The Discrete-Wavelet-Transform (DWT) is an indispensable tool in many applications requiring the processing of numerical data [1–3]. The power of the DWT lies in its ability to give a versatile multiresolution decomposition of the data it is analyzing [4,5]. The DWT is related to a two-channel multirate filter bank and is implemented using a tree-structured cascade of the basic two-channel system [6,7]. Wavelet and filter bank theories provide different ways of looking at and interpreting the signal decomposition that is being performed. With the traditional filter bank theory the decomposition is viewed as a frequency partitioning of a fullband signal into subbands. Wavelet theory views the decomposition as a partitioning into nested function spaces within a multiresolution framework. Both views are related and together provide a better understanding of such systems and this has spurred great interest in the theory, design and applications of such systems. Wavelet theory emphasizes the importance of vanishing moments (VM) and regularity of the wavelet functions which is something not considered in traditional filter bank theory. Wavelet filters can be classified as either biorthogonal or orthogonal. This paper will consider only orthogonal filters which have the advantages of noise decorrelation in denoising, simple bit-allocation in compression and more generally the l^2 norm (energy) preserving property.

The design of wavelet filter bank can be viewed as a constrained optimization problem. The first essential constraint is the perfect reconstruction (PR) or orthogonality constraint (some filter banks known as QMF [8] technically only have approximate PR). The second constraint is the VM constraint to ensure regularity in the equivalent wavelet functions. After imposing these constraints the remaining degrees of freedom can be optimized with respect to some chosen criterion which can be application specific. One criterion that has received at lot of attention from the community is the maximization of the compaction energy of a specific signal the wavelet is analyzing, i.e. signal adapted filter banks [9–12]. This compaction energy optimized filter has the potential of improved performance in applications such as signal analysis, coding and communications [11]. The frequency selectivity property of a traditional low-pass filter is however not ensured using this criterion. Good frequency selectivity is important to ensure the effects of aliasing or energy leakage is minimized in the subband signals and this is important in some applications such as system monitoring [13] and system identification in subbands [14]. Note that frequency selectivity only affects aliasing in the subbands and not the reconstructed signal (assuming no processing in the subbands). A perfect reconstruction filter bank has no aliasing in the reconstructed signal irrespective of the frequency selectivity as the aliasing in the subbands is canceled during reconstruction.

The challenge is therefore to design filters with good compaction energy while still having good frequency selectivity. This paper presents a method to design energy optimized filters that have a prescribed degree of transition band sharpness to ensure a desired degree of frequency selectivity. The method is based on the

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Zero-Pinning (ZP) technique on the Bernstein polynomial which is a simple and versatile technique for orthogonal wavelet filters. The ZP technique was first proposed in [15] where all the degrees of freedom were used to shape the frequency response of the filter by strategic pinning of the zeros of the polynomial. The ZP technique was then extended where only some of the degrees of freedom were used for pinning and the remaining degrees were used to optimize the filter with respect to the analytic quality (a criterion for the design of Hilbert-pairs) [16]. An exhaustive search based technique is required in the optimization in [16] because the objective function cannot be expressed as a convex function. The search technique is computationally inefficient and is only practical when the number of remaining degrees is small (less than two). In this paper only two degrees of freedom will be used for zero pinning and the remaining degrees will be used to optimize the filter with respect to some energy function which is convex. The optimization can be cast as a Semidefinite Programming (SDP) problem for which efficient algorithms and freeware which are widely available. With this approach both the frequency selectivity criterion and the compaction energy criterion can be simultaneously addressed in the filter design. The novelties of this paper are:

1. The design of wavelet filters using two criteria, i.e. compaction energy and frequency selectivity, at the same time.
2. Combining the ZP technique on the Bernstein polynomial with Semidefinite Programming in the design problem formulation.
3. Derivation of the objective function in terms of the Bernstein parameters, and the linear inequality bound on the filter response to ensure good frequency selectivity.

In Section 2 a review of the fundamentals of wavelets, filter banks and Bernstein polynomial as it relates to the work of this paper is presented. Section 3 describes the method for optimizing the filter with respect to an energy measure but with a prescribed degree of sharpness in the transition band. Relevant constraints to the problem are formulated here and it is shown how to cast the design problem as a semidefinite programming (SDP) problem. Design examples and discussions are presented in Section 4. Section 5 presents an application in image denoising and the paper concludes in Section 6.

2. Preliminaries and background

A two-channel multirate filter bank is made up of the following filters: $H_0(z)$ (low-pass analysis), $H_1(z)$ (high-pass analysis), $F_0(z)$ (low-pass synthesis) and $F_1(z)$ (high-pass synthesis). The following conditions must be satisfied to ensure perfect reconstruction (PR) [17]:

$$H_1(z) = z^{-1}F_0(-z), \qquad F_1(z) = zH_0(-z)$$

and

$$M(z) + M(-z) = 1 \tag{1}$$

where the product filter $M(z)$ is defined as

$$M(z) \equiv H_0(z)F_0(z)$$

For an orthogonal filter bank the low-pass filters are time-reverse versions of each other $F_0(z) = H_0(z^{-1})$. Define $H(z) \equiv H_0(z)$ (subscript 0 dropped) for convenience. The low-pass $H(z)$ filter is also known as a conjugate-quadrature-filter (CQF) and can be obtained from a spectral factorization of a product filter $M(z) = H(z)H(z^{-1})$. For convenience we shall use the almost-centered-at-the-origin (ACO) version of the CQF. A CQF impulse response $h(n)$, which must be of even length L_f, is said to be ACO if its impulse response support is given by $n \in [-(L_f/2-1), L_f/2]$. Using the ACO version the delay factor z^{-N} is not needed in the equations above. The product filter must satisfy the halfband constraint (1) and also the non-negativity constraint

$$M\left(e^{j\omega}\right) \geqslant 0 \tag{2}$$

The equivalent scaling function $\phi(t)$ and wavelet $\psi(t)$ are defined implicitly through the two-scale (refinement) equations:

$$\phi(t) = \frac{2}{H_0(1)} \sum_k h_0(k)\phi(2t-k)$$

$$\psi(t) = \frac{2}{H_1(1)} \sum_k h_1(k)\phi(2t-k)$$

where $h_0(k)$ and $h_1(k)$ are the coefficients of $H_0(z)$ and $H_1(z)$ respectively. Zeros at $z = -1$ are imposed on the CQF $H(z)$ to give vanishing moments (VM) to the equivalent wavelet $\psi(t)$ so that smoothness is achieved.

One way to construct the product filter is through the Parametric Bernstein Polynomial (PBP) introduced by Caglar and Akansu in [18]:

$$B_N(x;\boldsymbol{\alpha}) \equiv \sum_{i=0}^{N} f(i)\binom{N}{i}x^i(1-x)^{N-i} \tag{3}$$

where N is odd, $\boldsymbol{\alpha} = [\,\alpha_0\ \ldots\ \alpha_{(N-1)/2}\,]^T$ and

$$f(i) \equiv \begin{cases} 1-\alpha_i & 0 \leqslant i \leqslant \frac{1}{2}(N-1) \\ \alpha_{N-i} & \frac{1}{2}(N+1) \leqslant i \leqslant N \end{cases} \tag{4}$$

The z-transform product filter function can then be obtained as

$$M(z) = B\left(-\frac{1}{4}z\left(1-z^{-1}\right)^2\right)$$

where for brevity $B(x) = B_N(x;\boldsymbol{\alpha})$. The following substitution is used: $x = -\frac{1}{4}z(1-z^{-1})^2 = \sin^2(\frac{\omega}{2})$. The *PR constraint* (1) is automatically satisfied by the $M(z)$ constructed using the PBP. The desired number of zeros at $z = -1$ of $M(z)$ can be imposed by setting an appropriate number of Bernstein parameters to zero. Specifically setting $\alpha_i = 0$ for $i = 0, \ldots, L$ will give $2(L+1)$ zeros; hence $(L+1)$ zeros (vanishing moment) for the CQF $H(z)$. In summary the *PR constraint* and the *vanishing moment* (VM) condition are structurally guaranteed. This is the appeal of using the PBP. The set of non-zero Bernstein parameters:

$$\boldsymbol{\alpha}^{nz} \equiv [\alpha_{L+1},\ \ldots,\ \alpha_{(N-1)/2}]^T$$

can be regarded as free-parameters that can be used to tailor the characteristics of the filter to whatever desired criterion. The number of degrees of freedom available in the design process is:

$$d_f \equiv \dim\left(\boldsymbol{\alpha}^{nz}\right) = (N-1)/2 - L$$

3. Energy optimized filter with prescribed sharpness

Two commonly used energy measures will be considered in this work.

3.1. Least squares criterion

Consider first the traditional stopband energy measure of the filter given by [17]

$$\tilde{E}_S \equiv \int_{\omega_S}^{\pi} \left|H\left(e^{j\omega}\right)\right|^2 d\omega \tag{5}$$