Signal Processing 130 (2017) 37–46

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

DOA estimation exploiting a uniform linear array with multiple co-prime frequencies ☆

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ARTICLE INFO

Article history:
Received 17 November 2015
Received in revised form
24 May 2016
Accepted 8 June 2016
Available online 9 June 2016

Keywords:
DOA estimation
Co-prime array
Sparse sampling
Group sparsity
Sparse Bayesian learning

ABSTRACT

The co-prime array, which utilizes a co-prime pair of uniform linear sub-arrays, provides a systematical means for sparse array construction. By choosing two co-prime integers M and N, $O(MN)$ co-array elements can be formed from only $O(M+N)$ physical sensors. As such, a higher number of degrees-of-freedom (DOFs) is achieved, enabling direction-of-arrival (DOA) estimation of more targets than the number of physical sensors. In this paper, we propose an alternative structure to implement co-prime arrays. A single sparse uniform linear array is used to exploit two or more continuous-wave signals whose frequencies satisfy a co-prime relationship. This extends the co-prime array and filtering to a joint spatio-spectral domain, thereby achieving high flexibility in array structure design to meet system complexity constraints. The DOA estimation is obtained using group sparsity-based compressive sensing techniques. In particular, we use the recently developed complex multitask Bayesian compressive sensing for group sparse signal reconstruction. The achievable number of DOFs is derived for the two-frequency case, and an upper bound of the available DOFs is provided for multi-frequency scenarios. Simulation results demonstrate the effectiveness of the proposed technique and verify the analysis results.

1. Introduction

An important application of array signal processing is direction-of-arrival (DOA) estimation, which determines the spatial spectrum of the impinging electromagnetic waves. It is well known that an N-element uniform linear array (ULA) has $N-1$ degrees-of-freedom (DOFs), i.e., it resolves up to $N-1$ sources or targets by using conventional DOA estimation methods, such as MUSIC and ESPRIT [3,4]. On the other hand, a higher number of DOFs can be achieved to resolve more targets by using the same number of array sensors if they are sparsely placed [5,6]. An increased number of DOFs is usually achieved by exploiting the extended difference co-array whose virtual sensor positions are determined by the lag differences between the physical sensors.

Among a number of techniques that are available for sparse array construction, co-prime array [7] is considered attractive due to its capability of the systematic sparse array design. By choosing two integer numbers M and N to be co-prime, $O(MN)$ targets can be resolved with $M+N-1$ physical sensors [8]. This co-prime array concept can be generalized by introducing an integer factor that compresses the inter-element spacing of one constituting sub-array, thereby achieving increased DOFs [9,11]. In addition, by placing the two sub-arrays co-linearly instead of co-located, the number of unique virtual sensors is further increased, which benefits DOA estimation based on sparse signal reconstruction techniques [10,11].

While the co-prime array concept has been developed using physical uniform linear sub-arrays, we propose in this paper an effective scheme that implements co-prime array configurations using a single sparse ULA with two or more co-prime frequencies. As such, the ULA, whose inter-element spacing is respectively M_1 and M_2 half-wavelengths of the two respective frequencies, with M_1 and M_2 to be mutually co-prime integers, acts as virtual sub-arrays, resulting in an equivalent structure to co-prime arrays. In essence, the proposed approach integrates the concept of co-prime array and co-prime filter to reduce complexity and achieve high system performance. Unlike co-prime arrays, wherein the numbers of sub-array sensors and the inter-element spacings have to

☆The work of S. Qin, Y.D. Zhang, and M.G. Amin was supported in part by the Office of Naval Research under Grant No. N00014-13-1-0061 and by a subcontract with Defense Engineering Corporation for research sponsored by the Air Force Research Laboratory under Contract FA8650-12-D-1376. Part of the results was presented at IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, Saint Martin, December 2013 [1] and the SPIE Wireless Sensing, Localization, and Processing Conference, Baltimore, MD, May 2014 [2].
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http://dx.doi.org/10.1016/j.sigpro.2016.06.008
0165-1684/

satisfy the co-prime relationship, only the frequencies are required to be co-prime in the proposed scheme.

The proposed scheme can be adopted for both passive and active radar systems. The former requires filtering the signal arrivals at the employed co-prime frequencies, whereas the latter requires emitting those frequencies from a single antenna or a phased array and receiving the target backscattering with ULA. The transmitter and receiver can be located or widely separated. For active sensing, sum co-array of the transmit and receive arrays replaces the difference co-array of the two structures which is associated with receive only operations [12].

In this paper, we derive the analytical expression of the available number of DOFs as a function of the number of physical sensors, L, and the selected co-prime frequencies for the two-frequency case. The results resemble those derived in [9,11] for a physical co-prime array. The key difference lies in the fact that, unlike the co-prime array where each sub-array uses a different number of sensors, the two virtual sub-arrays in the underlying structure refer to the same physical ULA and thus share the same number of sensors. In addition, the number of physical sensors is not tied to the co-prime frequency multipliers M_1 and M_2. The property enables a higher flexibility in array design and operation. In particular, for a fixed number of physical array sensors, L, we demonstrate that a high number of DOFs, proportional to L^2, can be achieved with large values of M_1 and M_2. When K mutually co-prime frequencies are used, each pair of these frequencies can form a virtual co-prime array as discussed above. Accordingly, $O(K^2L^2)$ DOFs can be achieved.

It is shown that, in the proposed scheme, the self-lags in the co-array corresponding to each sub-array form a subset of the sub-array cross-lags. As such, the available DOFs are solely determined by the number of cross-lags between the two sub-arrays. Because of the frequency-dependent characteristics of the source, channel and target radar cross section (RCS), the received signal vectors corresponding to the different frequencies have a common spatial support, i.e., DOA, but generally have distinct coefficients. Thus, DOA estimations become a group sparse signal reconstruction problem. In this case, the self-lags obtained for each sub-array can also be exploited for possible performance improvement.

A large number of compressive sensing (CS) techniques have been proposed to deal with this problem. In this paper, we consider the problem under the Bayesian compressive sensing (BCS) or sparse Bayesian learning framework [13–17], which generally achieves a better reconstruction performance over those on the basis of the greedy algorithms and dynamic programming approaches, such as the orthogonal matching pursuit (OMP) [18] and the least absolute shrinkage and selection operator (LASSO) [19] algorithms. In particular, we use the complex multitask Bayesian compressive sensing (CMT-BCS) algorithm [20] to determine the DOAs of group sparse complex signals. This algorithm jointly treats the real and imaginary components of a complex value, in lieu of decomposing them into independent real and imaginary components. As a result, the sparsity of the estimated weight vectors can be improved, yielding better signal recovery. Group sparsity treatments for real and imaginary entries have been reported in, e.g., [21,22].

The remainder of the paper is organized as follows. In Section 2, we first review the co-prime array concept based on the difference co-array. Then, the array signal model exploiting co-prime frequencies is summarized in Section 3. Analytical expressions of array aperture and the number of DOFs are derived in Section 4 with respect to two and multiple co-prime frequencies. Sparsity-based DOA estimation exploiting the CMT-BCS is described in Section 5. Simulation results are provided in Section 6 to compare the performance of DOA estimation for different scenarios and validate the usefulness of the results presented in Section 5. Section 7 concludes this paper.

Notations: We use lower-case (upper-case) bold characters to denote vectors (matrices). In particular, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $(\cdot)^*$ implies complex conjugation, whereas $(\cdot)^T$ and $(\cdot)^H$ respectively denote the transpose and conjugate transpose of a matrix or vector. $\mathrm{vec}(\cdot)$ denotes the vectorization operator that turns a matrix into a vector by stacking all columns on top of each other, and $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix that uses the elements of $\mathbf{x}$ as its diagonal elements. $\|\cdot\|_2$ and $\|\cdot\|_1$ respectively denote the Euclidean (l_2) and l_1 norms, and $\mathrm{E}(\cdot)$ is the statistical expectation operator. $\otimes$ denotes the Kronecker product, and $\lfloor\cdot\rfloor$ denotes the floor function and returns the largest integer not exceeding the argument. $P_r(\cdot)$ denotes the probability density function (pdf), and $\mathcal{N}(x|a, b)$ denotes that random variable x follows a Gaussian distribution with mean a and variance b. $\mathrm{Re}(x)$ and $\mathrm{Im}(x)$ denote the real and imaginary parts of complex element x, respectively.

2. Co-prime array concept

In this section, we first review the co-prime array configuration that achieves a higher number of DOFs based on the difference co-array concept. A co-prime array [7] is illustrated in Fig. 1, where M and N are co-prime integers, i.e., their greatest common divisor is one. Without loss of generality, we assume $M < N$. The unit inter-element spacing d is typically set to $\lambda/2$, where λ denotes the wavelength. The array sensors are positioned at

$$\mathbb{P} = \{Mnd|\ 0 \le n \le N-1\} \cup \{Nmd|\ 0 \le m \le M-1\}. \tag{1}$$

Because the two sub-arrays share the first sensor at the zeroth position, the total number of sensors used in the co-prime array is $M + N - 1$. Note that the minimum inter-element spacing in this co-prime array is $d = \lambda/2$.

Denote $\mathbf{p} = [p_1, \ldots, p_{M+N-1}]^T$ as the positions of the array sensors, where $p_i \in \mathbb{P}$, $i = 1, \ldots, M+N-1$, and the first sensor, located at $p_1 = 0$, is assumed as the reference. Assume that Q uncorrelated signals impinging on the array from angles $\Theta = [\theta_1, \ldots, \theta_Q]^T$, and their discretized baseband waveforms are expressed as $s_q(t)$, $t = 1, \ldots, T$, for $q = 1, \ldots, Q$. Then, the data vector received at the co-prime array is expressed as,

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \mathbf{a}(\theta_q) s_q(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where

$$\mathbf{a}(\theta_q) = \left[1, e^{j\frac{2\pi p_2}{\lambda}\sin(\theta_q)}, \ldots, e^{j\frac{2\pi p_{M+N-1}}{\lambda}\sin(\theta_q)}\right]^T \tag{3}$$

is the steering vector of the array corresponding to θ_q, $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_Q)]$, and $\mathbf{s}(t) = [s_1(t), \ldots, s_Q(t)]^T$. The elements of the noise vector $\mathbf{n}(t)$ are assumed to be independent and identically distributed (i.i.d.) random variables following the complex Gaussian distribution $\mathcal{CN}(0, \sigma_n^2\mathbf{I}_{M+N-1})$.

The covariance matrix of the data vector $\mathbf{x}(t)$ is obtained as

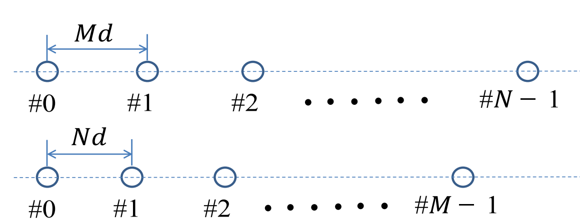

Fig. 1. The coprime array configuration.