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Efficient cable arrangement in cable stayed bridges based on sensitivity analysis of aeroelastic behaviour

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Abstract

Construction of cable supported bridges has experienced a great impulse in the past decade. Bridges having more than 800 m of span length were built in France and Japan and some bridges with span length of more than 1 km are going to be built, such as the Stonecuters bridge in Hong-Kong, and the Chongming in China (Fig. 1). Because of the increasing length of this class of bridges, they are becoming prone to phenomena like flutter in a similar way than long span suspension bridges.

Cable stayed bridges may present a few different alternatives for the cable system. At least harp, fan or modified fan arrangements can be discussed at the beginning of the design. Also variations in the number of cable planes can be studied.

Usually, during the design process changes are made by carrying out a number of analyses and using trial and error techniques relying in heuristic rules that are based upon the particular skills of the engineer. This approach can be inefficient in new problems and commonly it needs to be supported with results coming from experimental testing which makes more expensive the whole design process. Instead of that, an approach based in sensitivity analysis can be very helpful for the designer.

Keywords: Aeroelastic analysis; Efficient structural design; Cable stayed bridges; Cable configuration; Sensitivity analysis

1. Aeroelastic analysis of bridges

Wind forces per unit of deck length produced by a steady wind of speed U at any location of the bridge deck are usually represented by three components, D (drag), L (lift) and M (moment), see Fig. 2. The motion-induced forces may be expressed in terms of the displacement and velocities of the bridge deck using a linearized expression [5,2]:

$$\begin{pmatrix} D \\ L \\ M \end{pmatrix} = \frac{1}{2}\rho U^2 \begin{pmatrix} K^2P_4^* & K^2P_6^* & BK^2P_3^* \\ K^2H_6^* & K^2H_4^* & BK^2H_3^* \\ BK^2A_6^* & BK^2A_4^* & B^2K^2A_3^* \end{pmatrix} \begin{pmatrix} v \\ w \\ \varphi_x \end{pmatrix} + \frac{1}{2}\rho U^2 \begin{pmatrix} BKP_1^*/U & BKP_5^*/U & B^2KP_2^*/U \\ BKH_5^*/U & BKH_1^*/U & B^2KH_2^*/U \\ B^2KA_5^*/U & B^2KA_1^*/U & B^3KA_2^*/U \end{pmatrix} \begin{pmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{pmatrix} \quad (1)$$

where ρ is the air density, B is the reference width of the deck, K is the reduced frequency parameter $K = B\omega_D/U$, being ω_D the vibration frequency of the deck, and P_i^*, H_i^*, A_i^* $(i = 1, \ldots, 6)$ are aeroelastic coefficients (also called flutter derivatives) obtained by experimental testing and depending of the reduced frequency K. Therefore, forces in Eq. (1) correspond to the set of self-excited components of wind action and they are used for the flutter analysis.

At structural level, the complete set of motion-induced forces produced by wind flow can be expressed as:

$$\mathbf{P} = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} \quad (2)$$

where $\mathbf{P}$ is the vector of nodal wind forces, $\mathbf{C}_a$ and $\mathbf{K}_a$ are called aeroelastic damping and aeroelastic stiffness matrices and contain the overall contribution of the wind forces along the bridge deck.

Assuming that the structural behaviour is linear and viscous damping, the dynamic equilibrium of a structure is written as

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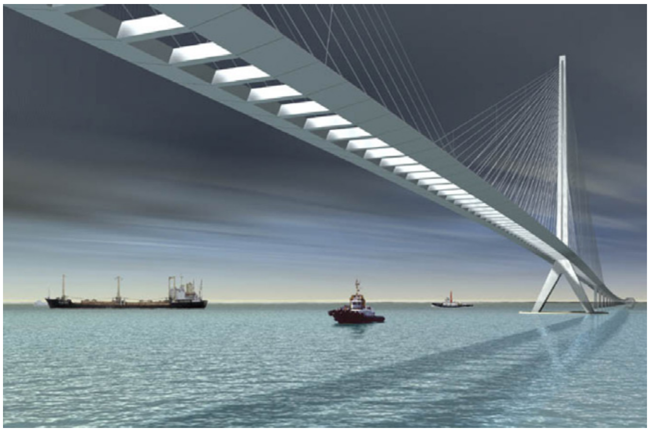

Fig. 1. Chongming Bridge (Halcrow Group Limited Flint & Neill Partnership).

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{P} = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} \tag{3}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the mass, damping and stiffness matrices. Reorganizing expression (3)

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{C} - \mathbf{C}_a)\dot{\mathbf{u}} + (\mathbf{K} - \mathbf{K}_a)\mathbf{u} = \mathbf{0} \tag{4}$$

Eq. (4) represents free vibration of a structural system, which can be solved by modal analysis considering m vibration modes

$$\mathbf{u} = \sum_{r=1}^{m} \boldsymbol{\phi}_r q_r = \boldsymbol{\Phi}\mathbf{q} \tag{5}$$

where $\boldsymbol{\sigma}$ is the modal matrix, and vector $\mathbf{q}$ is formulated by

$$\mathbf{q} = \mathbf{w}e^{\mu t} \tag{6}$$

Substituting in (4), premultiplying by $\boldsymbol{\sigma}^{\mathrm{T}}$, defining matrices through (7) and normalizing eigenvectors with respect to $\mathbf{M}$

$$\mathbf{M}_{\mathrm{R}} = \mathbf{I} = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M}\boldsymbol{\Phi}, \quad \mathbf{C}_{\mathrm{R}} = \boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{C} - \mathbf{C}_a)\boldsymbol{\Phi}, \quad \mathbf{K}_{\mathrm{R}} = \boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{K} - \mathbf{K}_a)\boldsymbol{\Phi} \tag{7}$$

It turns out that expression (4) becomes

$$(\mu^2\mathbf{I}\mathbf{w} + \mu\mathbf{C}_{\mathrm{R}}\mathbf{w} + \mathbf{K}_{\mathrm{R}}\mathbf{w})e^{\mu t} = \mathbf{0} \tag{8}$$

which represents an eigenvalue problem. To obtain the solution, the following identity is included in the formulation

$$-\mu\mathbf{I}\mathbf{w} + mu\mathbf{I}\mathbf{w} = \mathbf{0} \tag{9}$$

Combining expressions (8) and (9)

$$\left(\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mu^2\mathbf{w} \\ \mu\mathbf{w} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{\mathrm{R}} & \mathbf{K}_{\mathrm{R}} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mu\mathbf{w} \\ \mathbf{w} \end{bmatrix} \right) e^{\mu t} = \mathbf{0} \tag{10}$$

If the following terms are defined

$$\mathbf{w}_\mu = \begin{pmatrix} \mu\mathbf{w} \\ \mathbf{w} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} -\mathbf{C}_{\mathrm{R}} & -\mathbf{K}_{\mathrm{R}} \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \tag{11}$$

Eq. (10) becomes

$$(\mathbf{A} - \mu\mathbf{I})\mathbf{w}_\mu e^{\mu t} = \mathbf{0} \tag{12}$$

The eigenvalue problem defined by (12) provides a set of pairs of conjugate complex eigenvalues μ_i and $\bar{\mu}_i$

$$\mu_i = \alpha_i + i\beta_i, \quad \bar{\mu}_i = \alpha_i - i\beta_i \tag{13}$$

An eigenvalue, alongside the eigenvector associated with it, will define the shape of the damping oscillation of the deck, expressed as:

$$\begin{aligned} \mathbf{u} = {} & \boldsymbol{\Phi}[\mathbf{w}_{\mathrm{R}} \cos(\beta_i t) - \mathbf{w}_{\mathrm{I}} \operatorname{sen}(\beta_i t)]e^{\alpha_i t} \\ & + i\boldsymbol{\Phi}[\mathbf{w}_{\mathrm{R}} \operatorname{sen}(\beta_i t) + i\mathbf{w}_{\mathrm{I}} \cos(\beta_i t)]e^{\alpha_i t} \end{aligned} \tag{14}$$

From Eq. (14), it can be observed that the imaginary part of the eigenvalues counts on the aeroelastic influenced frequency $\omega_{a,i} = \beta_i$ while the real part of the eigenvalues is associated with the aeroelastic influenced damping ratio by means of $\alpha_i = -\xi_a\omega$, where ω is the natural frequency.

2. Solution of the flutter problem

With moderate wind speeds U every aeroelastic damping ratio ζ_{ai} will be positive and, therefore, structural response

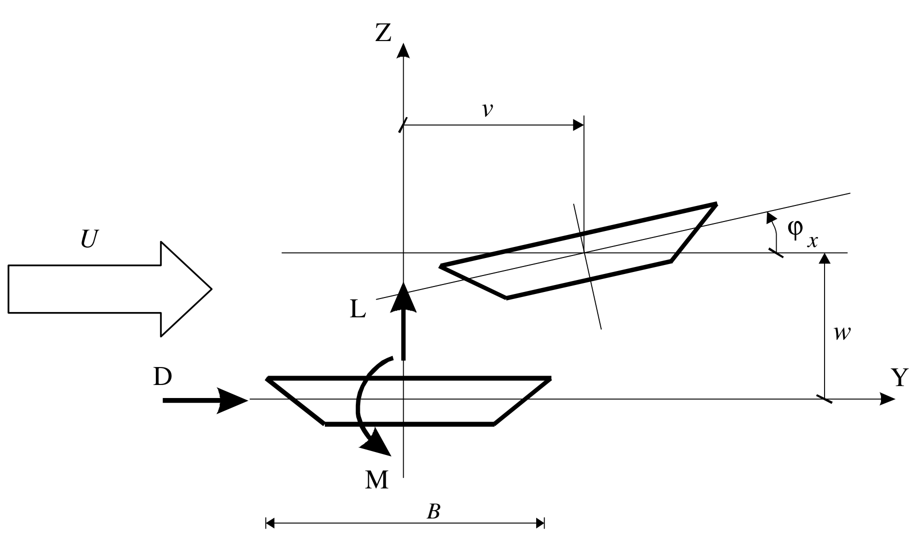

Fig. 2. Aeroelastic forces on a bridge deck.