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Another look on the structure of mountain waves: A spectral perspective

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ABSTRACT

Linear wave solutions in the spectral space are analyzed to help understand the structure of mountain waves. Nonrotating and hydrostatic waves generated in wind with directional shear past a circular bell-shaped mountain are studied. The power spectra of perturbed vertical velocity and pressure are symmetrically distributed about the orientation of dominant wave component, which bisects the angle between surface wind and local wind directions. The maximum power spectrum increases with the horizontal wind speed but decreases with the wind turning angle. The power spectra of potential temperature and horizontal velocity exhibit an asymmetric distribution except at the surface, which are infinite for the wave components normal to the mean wind. These large-amplitude perturbations of potential temperature and horizontal velocity are advected downstream and the waves finally break, giving rise to the occurrence of turbulent wakes at various heights. All the wavefields rotate with height in the same direction of the mean wind. However, the perturbed vertical velocity and pressure turn at a rate slower than that of horizontal velocity and potential temperature. The application of spectral analysis to the wave momentum flux is discussed, which helps explain the misalignment of wave momentum flux with the surface wind.

1. Introduction

Orographic gravity waves (OGWs) are excited as stably stratified flows interact with mountains on the earth. OGWs are capable of vertically transporting momentum from their source (i.e., orography) to the upper atmosphere, thus exerting an important influence on the mesospheric general circulation (Holton, 1983). Mountain wave activities are also intimately related to many weather phenomena, some of which may cause casualties. For example, the breaking of OGWs is an importance source of clear air turbulence which threatens the safety of aviation. Wildfire may be induced in the leeside of high mountains through the well-known foehn effect (Smith, 1985).

In the theoretical studies of OGWs, the wave equation is often solved in the spectral space, which is an ordinary differential equation. The spectral solution is then remapped to the physical space to examine the mountain wave structure. For example, Smith (1980) investigated the structure of nonrotating hydrostatic waves generated in constant wind over a three-dimensional (3D) circular bell-shaped mountain. While the remapping of spectral wave solution to the physical space offers many advantages, it has shortcomings as well. The remapped solution in the physical space often contains integrals that cannot be represented by elementary functions. As a result, they must be computed numerically (e.g., Teixeira and Miranda, 2006; Xu et al., 2012, hereafter XWX12). It is not easy to understand the underlying physics according to complex integral expressions.

Since the physical solution can be viewed as a superposition of different Fourier harmonics, it is natural to study the spectral solution directly, i.e., spectral analysis. This spectral perspective has been adopted in the study of gravity wave momentum flux (WMF), which is closely related to the parameterization of gravity wave drag in numerical models (Kim et al., 2003). For instance, McFarlane (1987) parameterized the orographic WMF using a single wave along the surface wind direction. In contrast, Hines (1988) found that it was more appropriate to represent the WMF from isotropic terrain using two waves rather than a single one. By analyzing the WMF spectrum at the cloud top, Song and Chun (2005) developed a spectral parameterization scheme for non-orographic gravity wave drag.

In this work, the mountain wave structure generated in directionally sheared winds past a circular bell-shaped mountain is studied as an example from the spectral perspective. This kind of flow has already been investigated by Shutts (1998) through a combination of ray-tracing and stationary-phase methods. The wavefield was found to broaden with height, similar to the constant wind case (Smith, 1980). Moreover, as the ambient wind turns with height, the wavefield also rotates in the same direction as the wind but at a relatively slower rate. The presence

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of directional wind shear causes a number of selective critical levels, at which the horizontal wave vector is normal to the local mean wind (Broad, 1995; Shutts, 1995). Mountain waves are advected downstream on meeting the selective critical levels, which finally break and in consequence produce turbulent wakes at various altitudes (Broad, 1999). The stationary-phase ray solution in Shutts (1998) has a simple form with no integral. Yet it is only valid far away from the wave source (i.e., mountain). In view of this, the Maslov's method was suggested in Broutman et al. (2002), which can avoid the ray-solution singularity directly above the terrain. However, the Maslov solution consists of an integral expression. As will be shown in this study, the OGW structural features can be readily understood from the viewpoint of spectral analysis.

The rest of this paper is organized as follows. Section 2 derives the spectral wave solutions for hydrostatic nonrotating mountain waves generated in directional wind varying linearly with height. The spectral solutions are analyzed in Section 3 for a particular case of circular bell-shaped mountain. The application of spectral analysis to the WMF is presented in Section 4. Finally, Section 5 summarizes this paper with discussions.

2. Linear wave solution

According to the theory of two-dimensional (2D) Fourier transforms, the vertical velocity in the spectral space is

$$\hat{w}(k,l,z) = (2\pi)^{-2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w(x,y,z)\ \exp[-i(kx+ly)]dxdy, \quad (1)$$

where $\mathbf{K} = (k, l)$ is the horizontal wave vector, and $w(x, y, z)$ is the vertical velocity in the physical domain. Under the assumption of steady, adiabatic, inviscid, nonrotating, hydrostatic, and Boussinesq flow, the governing equation of spectral vertical velocity is given by (cf. Eq. (9) of XWX12),

$$\hat{w}_{zz}z + m(z)^2\hat{w} = 0, \quad (2)$$

where $m^2 = N^2K^2/\hat{D}(z)^2$ is the squared vertical wave number, $K = |\mathbf{K}|$, N is the Brunt-Väisälä frequency, and $\hat{D}(z) = \mathbf{V}(z) \cdot \mathbf{K}$. (Hereafter, the subscript represents partial derivative unless otherwise stated or defined.) In deriving the above equation, the horizontally uniform mean wind is assumed to vary *linearly* with height, i.e.,

$$\mathbf{V}(z) = (U_0 + U_z z, V_0 + V_z z) = |\mathbf{V}(z)|(cos\psi(z), sin\psi(z)), \quad (3)$$

where $\mathbf{V}_0 = (U_0, V_0)$ is the surface wind, $\mathbf{V}_z = (U_z, V_z)$ the vertical shear, and $\psi(z)$ is the azimuth of $\mathbf{V}(z)$. By virtue of the Frobenius method (e.g., Booker and Bretherton, 1967), the analytical solution of the wave equation is (see XWX12)

$$\hat{w}(K,\varphi,z) = \hat{w}(K,\varphi,0)\sqrt{\frac{\hat{D}(z)}{\hat{D}_0}}\ \exp\left[\text{isgn}\left(\hat{D}_z\right)\ \ln\frac{\hat{D}(z)}{\hat{D}_0}\sqrt{\frac{\text{Ri}}{\cos^2(\chi_0-\varphi)}-\frac{1}{4}}\right], \quad (4)$$

where φ is the azimuth of $\mathbf{K}$, namely, $\mathbf{K} = K(\cos\varphi, \sin\varphi)$, $\hat{D}_0 = \mathbf{V}_0 \cdot \mathbf{K}$, $\hat{D}_z = \mathbf{V}_z \cdot \mathbf{K}$, sgn(·) is the sign function, $\text{Ri} = N^2/|\mathbf{V}_z|^2$ is the mean flow Richardson number, and χ_0 is the azimuth of wind vertical shear. At the lower boundary, it satisfies the free-slip condition $\hat{w}(k, l, 0) = i\hat{D}_0\hat{h}$, where $\hat{h} = \hat{h}(K, \varphi)$ is the 2D Fourier transform of the mountain $h(x, y)$. Note that wave transmission above critical levels is excluded by assuming that gravity waves are totally absorbed at critical levels in the case of large Richardson numbers (Booker and Bretherton, 1967).

Once the vertical velocity is obtained, one can readily derive other wave variables, such as horizontal velocity, pressure, and potential temperature

$$\hat{u} = \frac{i}{K^2}\left(k\frac{\partial}{\partial z} - l\frac{kV_z - lU_z}{\hat{D}}\right)\hat{w}, \quad (5)$$

$$\hat{u} = \frac{i}{K^2}\left(l\frac{\partial}{\partial z} + k\frac{kV_z - lU_z}{\hat{D}}\right)\hat{w}, \quad (6)$$

$$\hat{p} = i\frac{\overline{p}}{K^2}\left(\hat{D} - \hat{D}\frac{\partial}{\partial z}\right)\hat{w}, \quad (7)$$

$$\hat{\theta} = \frac{i}{\hat{D}}\frac{\partial\overline{\theta}}{\partial z}\hat{w} \quad (8)$$

where $\overline{\rho}$ is the reference density, and $\overline{\theta}(z)$ is the base-sate potential temperature. These equations give the relative phases and amplitudes of different wave quantities, known as polarization relation (see Appendix A).

3. Results

3.1. Spectral analysis

In accordance with Eqs. (5)–(8), the power spectra of perturbed vertical velocity, pressure, potential temperature, and horizontal velocity are given by,

$$|\hat{w}(K,\varphi,z)|^2 = |\mathbf{V}_0||\mathbf{V}(z)|\ cos(\varphi-\psi_0)\ cos[\varphi-\psi(z)]K^2\left|\hat{h}\right|^2, \quad (9)$$

$$|\hat{p}(K,\varphi,z)|^2 = \hat{p}^2\frac{N^2}{K^2}\left|\hat{w}(K,\varphi,z)\right|^2, \quad (10)$$

$$|\hat{\theta}(K,\varphi,z)|^2 = \left(\frac{\partial\overline{\theta}}{\partial z}\right)^2\frac{\hat{D}_o}{\hat{D}(z)}\left|\hat{h}\right|^2, \quad (11)$$

$$|\hat{u}(K,\varphi,z)|^2 = |\mathbf{V}_z|^2\frac{\hat{D}_o}{\hat{D}(z)}\left[Ricos^2\varphi + \ cos\chi_0 sin\varphi sin(\varphi-\chi_0)\right]\left|\hat{h}\right|^2, \quad (12)$$

$$|\hat{u}(K,\varphi,z)|^2 = |\mathbf{V}_z|^2\frac{\hat{D}_o}{\hat{D}(z)}\left[Risin^2\varphi - sin\chi_0\ cos\varphi sin(\varphi-\chi_0)\right]\left|\hat{h}\right|^2, \quad (13)$$

To facilitate the spectral analysis, we use a circular bell-shaped orography, i.e.,

$$h(x,y) = h_m\left[1 + (x/r_a)^2 + (y/r_a)^2\right]^{-3/2}, \quad (14)$$

where h_m and r_a are the mountain height and half width, respectively. (It should be noted that the results obtained below are valid for any *axisymmetric* mountain.) The spectrum of the isotropic bell-shaped mountain only depends on the horizontal wave vector magnitude

$$\hat{h}(K) = (2\pi)^{-1}h_m a^2 e^{-r_a K}. \quad (15)$$

In this case, the partial derivative of $|\hat{w}|^2$ with respect to φ (namely, the orientation of horizontal wave vector) is

$$\frac{\partial}{\partial\varphi}\left(|\hat{w}(K,\varphi,z)^2|\right) = -\left|\mathbf{V}_0||\mathbf{V}(z)|K^2|\hat{h}|^2 sin^2[2\varphi-\psi_0-\psi(z)]. \quad (16)$$

Evidently, Eq. (16) equals to zero at

$$\varphi_{max}(z) = [\psi_0 + \psi(z)]/2, \quad (17)$$