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# Bivariate Taylor-series expansion method of moment for particle population balance equation in Brownian Coagulation

JiangZhi[a], ShenJie[a], LuZhiming[a,*]

[a]*Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China*

**Abstract**

In this study, we extend the Taylor-series expansion method of moment to two-component aggregation problem undergoing Brownian coagulation with kernels that are independent of composition. A set of closed particle population balance equation for lower-order moments is then derived. Numerical results and its asymptotic solutions are validated by comparing with Monte Carlo simulation method both in free molecular regime and continuum regime. It is shown that three dimensionless particle moments $M_{C1}, M_{C2}, M_{C3}$ almost approach to a same value over large evolution time. The normalized variance of excess component A decreases as $1/\overline{v}$ and it tends to zero over large evolution time.

*Keywords:* moment methods; two-component; Brownian coagulation; excess component A

## 1. Introduction

Atmospheric aerosol particles have been a common environmental pollution (Harrison et al., 2000; Wiseman et al., 2009). The particle population balance equation (PBE) is a well-known model taken to describe the evolution of particle coagulation. A large number of researches have been done to the problem of aggregation in single-component systems (McGraw et al., 1997; McCoy et al., 2003; Yu et al., 2008; Xie et al., 2013). However, in many applications of practical interest particles are multi-component, such as: granulated pharmaceutical powders (Iveson et al., 2002), metal-oxide particles (Kuang et al., 1998), and crystallization (Puel et al., 2003). These circumstances require particles to be identified with two or more of their attributes, such as mass for two or more different compositions, mass and surface area, mass of primary particles and binder volume, particle volume, and uncapped surface area. In most cases,the coagulation kernel is a function of both size and composition of the particles. Some work has been done for this important case. The first analytical solution of Smoluchowski's two-component equation for kernel independent(constant kernel) of composition of colliding particles was solved by Lushnikov (1976) in discrete variant, and showed that the compositional distribution is given by binomial distribution, if the

*Corresponding author
*Email address:* zmlu@shu.edu.cn (LuZhiming)