Water Research 111 (2017) 163–176

Contents lists available at ScienceDirect

Water Research

journal homepage: www.elsevier.com/locate/watres

General combined model for the hydrodynamic behaviour of fixed and fluidised granular beds

Garry Hoyland

Bluewater Bio Ltd, Winchester House, 259-269 Old Marylebone Road, London, NW1 5RA, UK

A R T I C L E I N F O

Article history:
Received 19 July 2016
Received in revised form
9 December 2016
Accepted 2 January 2017
Available online 3 January 2017

Keywords:
Blake
Kozeny
Fixed
Fluidised
Wall
Model

A B S T R A C T

This work describes the derivation of a general mathematical model applicable to both fixed and fluidised granular beds, operating within the full hydrodynamic spectrum from viscous to inertial flows. The fundamental insight for the derivation of the model is that practical fluidised beds and fixed beds have similar hydrodynamic properties. The validity of the general model is demonstrated for fluid fractions up to 0.90. A crucial development in the general model is the replacement of hydraulic diameter, which has served as the size descriptor of flow paths in most fixed-bed models derived since the advent of the classic Blake-Kozeny equation. The new, replacement expression is based on the physical structure of the cross section of random porous beds. In addition, the general model contains a tortuosity factor, derived from the results of previous works involving computational fluid dynamics, to correct flow path length and fluid velocity.

The model is constructed using regression analysis of experimental data from six previous major works and tested against previous models.

1. Introduction

The work described here stems from a need to improve further the understanding of the hydrodynamic principles relating to water flow through granular filter beds operating in fixed mode and regenerative, fluidised mode. This has led to the development of a General Combined Model, GCM, applicable to both fixed and fluidised modes operating over the full hydrodynamic spectrum.

The GCM relates to fixed and fluidised packings comprising smooth, solid and spherical or near-spherical granules that have a similar packing geometry. Without further development, the GCM is not applicable to packings comprising other particle shapes such as cylindrical pipes and prisms.

1.1. Contents

Section 1.2 describes historical models derived for fixed and fluidised beds, relevant to this work. The GCM has been developed from conventional hydrodynamic principles, which are described in Chapter 2. The prime development of this work, described in Chapter 3, relates to the derivation of theoretical relationships that characterise (i) the magnitude of the interstitial velocity, U, of the fluid passing through convoluted flow paths in random porous networks of fixed and fluidised beds and (ii) the physical size of the cross-sectional area of the flow paths. The same relationships apply to both types of beds, over the practical range of fluid fractions, E. As shown in Chapter 6, the GCM is applicable to E values $\leq$ 0.90. Chapter 4 tests the validity of derived sensitivities of pressure loss to E in fixed beds and fluid velocity to E in fluidised beds, using previously published experimental data. Chapter 5 constructs the GCM and uses previously published data from several major sources to determine the values of coefficients. Chapter 6 compares GCM predictions of the relationships between pressure loss and Reynolds number in fixed beds and between E and Reynolds number in fluidised beds with experimental data published in one of the major sources.

1.2. Historical models

Historical models for fixed and fluidised beds have largely been developed in parallel, leading currently to separate models with different structures for each bed type. An exception is the modelling of Foscolo et al. (1983) who developed models for fixed and fluidised beds using a common methodology. The following subsection discusses a selection of models relevant to this work.

E-mail address: garry.hoyland@gmail.com.

http://dx.doi.org/10.1016/j.watres.2017.01.008
0043-1354/

Nomenclature

Dimensional symbols

A_x (m^2)	area of flow stream's cross-section
A_b (m^2)	total cross-sectional area of bed
A_w (m^2)	the total wetted wall area of a conduit
C	a context-dependent dimensioned or dimensionless constant
D_c (m)	diameter of pipe.
D_x (m)	hydraulic diameter equal to A_x/L_w
D_m (m)	diameter of sphere, the volume of which V_m
D_v (m)	diameter of bed
L (m)	path length of flow stream
L_0 (m)	longitudinal length of fixed bed
L_b (m)	longitudinal length of fixed or expanded bed, equal to L_0 for fixed bed
L_w (m)	wetted perimeter of a flow stream's cross-section
L_x (m)	characteristic dimension of the cross-section of a conduit
L_u (m)	side length of cubic packing unit
S_m (m^2/m^3)	the total surface area of granules divided by total volume of granules
S_b (m^2/m^3)	total specific surface area of a bed, including area of vessel wall
S_v (m^2/m^3)	specific surface area of vessel
U (m/s)	average velocity along the axis of a conduit, or average interstitial velocity along the axis of a convoluted conduit in a bed
U_b (m/s)	superficial (empty-bed) velocity through bed
U_f (m/s)	free settling velocity of granules
V_m (m^3)	total volume of granules in a bed divided by the number of granules
V_u (m^3)	volume of a cubic packing unit
V_v (m^3)	total volume of the conduit or bed

Dimensionless symbols

Ar	Archimedes number
CI_{95}	95-percentile, multiplicative, confidence interval
De	density number equal to $(\rho_m - \rho)/\rho$ and Ar/Ga
E_0	fluid fraction of fixed bed
E	overall fluid fraction of bed, equal to E_0 for fixed bed
f( )	context-dependent function
Ga	Galilei number
$Ga_\#$	a modifiied form of Galilei number
K	a general hydraulic constant, with a value dependent on context
K_l	the hydraulic constant for viscous flow, with a value dependent on context
K_t	the hydraulic constant for inertial flow, with a value dependent on context
Q	coefficient in Felice and Kehlenbeck equation
R_d	diameter ratio equal to D_m/D_v
Re	Reynolds number equal to $L_x\, U\, \rho/\mu$
Re_f	Reynolds number equal to $D_m\, U_f\, \rho/\mu$
Re_m	Reynolds number equal to $D_m\, U_b\, \rho/\mu$
Re_s	Reynolds number equal to $U_b\, \rho/(\mu\, S_m(1-E))$
Re_d	Reynolds number equal to $D_m\, U_b\, \rho/(\mu\,(1-E))$
T	tortuosity of flow path through a granular bed

Greek symbols

Θ	friction factor derived by Blake for fixed beds
Θ_d	analogous to Θ, with S_m replaced with D_m
Φ	friction factor in GCM for fixed and fluidised beds
Δp (Pa)	loss in dynamic pressure over path length
Δh (m)	loss in head of fluid over path length, equal to $\Delta p/(\mathbf{g}\,\rho)$
μ (Pa s)	viscosity of fluid
ψ	sphericity equal to ratio of surface area of sphere to that of a granule where the sphere and granule have the same volume
ρ (kg/m^3)	density of fluid
ρ_m (kg/m^3)	material density of granules
τ (s)	retention time of fluid in bed
ω	the wall-effect factor derived by Carman
ω_a	the wall-effect factor in the fixed component of the GCM
ω_b	the wall-effect factor in the fluidised component of the GCM

ω_1, ω_2 and ω_3 are context-sensitive wall-effect factors

Exponents

m	exponent in GCM, dependent on mode of bed
n	the expansion exponent in the Richardson and Zaki fluidisation model, when $R_d = 0$
N	the expansion exponent in the Richardson and Zaki fluidisation model, when $R_d > 0$
x	context-dependent exponent
y	context-dependent exponent in Equations (4.5) and (4.6)

1.2.1. Fixed beds

A major advance in the modelling of fixed beds containing granules and other media was made by Blake (1922) who applied conventional hydrodynamics principles to porous beds. Further described in Chapter 3, his model is:

$$\Theta = K\, Re_s^{-x} \tag{1.1}$$

where

$$\Theta = \Delta p\, E^3 \Big/ \left(L_b\, \rho\, U_b^2\, S_m(1-E)\right) \tag{1.2}$$

and

$$Re_s = K\, U_b\, \rho/(\mu\, S_m(1-E)\,). \tag{1.3}$$

The dimensionless variables, Θ, K and Re_s are friction factor, empirical hydrodynamic constant and a Reynolds number respectively. Re_s is occasionally referred to as the Blake number. When the x exponent = 0.2, Equation (1.1) gives a reasonable fit to experimental data over a limited range of Re_m values of from 0.1 to 4. Making an analogy with flow through pipes, the x value is directly related to Reynolds number, decreasing from a theoretical value of 1 for viscous flow to a value of 0 for inertial flow.

Kozeny (1927) independently derived a form of Equation (1.1) specifically for viscous flow. His equation may also be obtained by substituting 1 for the exponent, x, in Blake's equation giving:

$$\Delta p \,/\, L_b = K_l\, \mu\, U_b\, S_m^2 (1-E)^2 \Big/ E^3. \tag{1.4}$$

Kozeny (1927) and Donat (1929) showed experimentally that