A Joint-optimization NLMS Algorithm with Linear Function Approximation Penalty for Sparse Channel Estimation

Yanyan Wang[1], **Yingsong Li**[1,2], **and Fanping Shi**[1]

[1]College of Information and Communications Engineering, Harbin Engineering University, Harbin, China
[2]National Space Science Center, Chinese Academy of Sciences, Beijing 100190, China

Abstract— In this paper, a linear function approximation penalty is introduced into the joint-optimization normalized least mean square (JO-NLMS) algorithm to utilize the sparseness of the multi-path channels in the hilly terrain communication environment. The proposed algorithm is denoted as linear function approximation (LFA) penalty JO-NLMS algorithm (LFAJO-NLMS), which is realized by integrating the LFA into the JO-NLMS's cost function. The proposed LFAJO-NLMS algorithm is mathematically given in detail and its channel estimation behaviors are confirmed over a multi-path model. The computer simulation results gotten from the sparse channel estimation show that our developed LFAJO-NLMS algorithm performs better than the early reported NLMS and its corresponding sparse versions.

1. INTRODUCTION

So far, adaptive filter algorithms have been widely used in the estimation of sparse multi-path channels [1]. The most typical least mean square (LMS) algorithm has been deeply discussed in sparse channel estimation owing to its low computational complexity and fast convergence speed [1, 2]. However, the LMS cannot achieve an optimal solution for sparse channel estimation in low signal-to-noise ratios (SNR) environment. In order to solve this problem, a normalizing of the input signal power is introduced into the typical LMS to develop the normalized LMS (NLMS) algorithm [3, 4]. Although the NLMS can effectively implement the channel estimation, its performance is not very well and it cannot exploit the sparsity of sparse channels.

To utilize the inherent sparsity characteristic of multi-path channels, some LMS and NLMS algorithms with sparsity functions are developed by incorporating norms into the cost function of the LMS and NLMS algorithms, which are motived by the compressed sensing (CS) techniques [4–17]. As is well-known, the zero attracting LMS (ZA-LMS) is obtained by introducing a l_1-norm into the LMS's cost function. The ZA-LMS can obtain a better estimation performance by the designed zero attractor using l_1-norm than the conventional LMS algorithm [7]. Then, a reweighting factor is introduced into the ZA-LMS algorithm to develop reweighting (RZA-LMS) algorithm [7]. Similarly, the zero attracting NLMS (ZA-NLMS) and reweighting ZA-NLMS (RZA-NLMS) algorithms have also been exploited based on the norm penalty. As a result, they can get a very well estimation performance for sparse channel estimation [15–17]. However, the performance of sparse NLMS algorithms may be limited when the step-size and regularized parameters are fixed. Recently, the joint-optimization method [17] is presented to remedy the shortcoming of the sparse NLMS algorithms. Then, the zero-attracting joint-optimization NLMS (ZAJO-NLMS) and reweighted ZAJONLMS (RZAJO-NLMS) algorithms are developed and discussed in [18, 19]. The ZAJO-NLMS and RZAJO-NLMS can achieve a good estimation performance with respect to steady-state error and convergence. However, we known that the ZAJO-NLMS use an uniform l_1 penalty on all of the coefficients, which may degrade the estimation performance.

In this paper, a linear function approximation penalty is introduced into the joint-optimization normalized LMS (JO-NLMS) algorithm to design a fire-new zero attractor, which can avoid employ an uniform penalty on all the coefficients [18, 19]. The proposed algorithm is denoted as linear function approximation (LFA) penalty JO-NLMS algorithm (LFAJO-NLMS) that is investigated under multi-path channels in the hilly terrain communication environment. The simulation results show that the proposed LFAJO-NLMS gains a superior performance than that of the ZAJO-NLMS and RZAJO-NLMS algorithms.

2. THE NORM PENALIZED JO-NLMS ALGORITHMS

In sparse channel estimation framework, the unknown signal $\mathbf{g} = [g_1, g_2, \ldots, g_M]^T$ is estimated by input signal $\mathbf{x}(n)$ and expected signal $d(n) = \mathbf{g}^T\mathbf{x}(n) + r(n)$ to minimize the error signal $e(n)$. Here, M denotes the length of the sparse channel, and $r(n)$ is an additional white Gaussian noise

(AWGN). The error signal is defined as $e(n) = d(n) - \hat{\mathbf{g}}^T(n)\mathbf{x}(n)$, where $\hat{\mathbf{g}}(n)$ is estimated signal. The updated euqation of the ZAJO-NLMS algorithm is expressed as [18]

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \frac{\left[m(n-1) + M\delta_n^2\right] e(n)\,\mathbf{x}(n)}{M\delta_r^2 + (M+2)\,\delta_x^2\left[m(n-1) + M\delta_n^2\right]} - \rho_{\mathrm{ZAJO}}\mathrm{sgn}\left(\hat{\mathbf{g}}(n)\right), \tag{1}$$

where, δ_r^2 and δ_x^2 represent noise power and the variance of $\mathbf{x}(n)$, respectively. δ_n^2 is a variance that give an important uncertainty on $\mathbf{g}$, and $m(n)$ is defined as

$$m(n) = \left\{1 - \frac{\delta_x^2\left[m(n-1) + M\delta_n^2\right]}{M\delta_r^2 + (M+2)\,\delta_x^2\left[m(n-1) + M\delta_n^2\right]}\right\}\left[m(n-1) + M\delta_n^2\right]. \tag{2}$$

When $n \to \infty$, it has

$$\delta_x^2\left[m(\infty) + M\delta_n^2\right] - M(M+2)\,\delta_n^2\delta_x^2\left[m(\infty) + M\delta_n^2\right] - M^2\delta_n^2\delta_r^2 = 0, \tag{3}$$

whose solution is written as

$$m(\infty) = \frac{M\delta_n^2}{2}\left[M + (M+2)\sqrt{1 + \frac{4\delta_r^2}{(M+2)^2\delta_n^2\delta_x^2}}\right]. \tag{4}$$

Similar with the ZA-LMS algorithm [7], a reweighting factor ε is introduced into the ZAJO-NLMS to develop the reweighting ZAJO-NLMS (RZAJO-NLMS) algorithm. Thus, the updating equation is achieved

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \frac{\left[m(n-1) + N\delta_n^2\right] e(n)\,\mathbf{x}(n)}{M\delta_r^2 + (M+2)\,\delta_x^2\left[m(n-1) + N\delta_n^2\right]} - \gamma_{\mathrm{RZAJO}}\frac{\mathrm{sgn}\left(\hat{\mathbf{g}}(n)\right)}{1 + \varepsilon\left|\hat{\mathbf{g}}(n)\right|}. \tag{5}$$

From the updating equation of ZAJO-NLMS and RZAJO-NLMS algorithms, it can find that the step-size and regularized parameter are joint-optimized when the zero attractor keeps invariable. However, the zero attractor provided by l_1-norm is limited to develop the sparsity of the multi-path channel because it gives an uniform penalty on all of the coefficients.

3. PROPOSED LFAJO-NLMS ALGORITHM

In this section, we develop an optimal zero attractor term using a linear function approximation penalty to improve the above mentioned drawback. The linear function approximation (LFA) penalty is defined as

$$S_\beta\left(\hat{\mathbf{g}}(n)\right) = \left(1 + \beta^{-1}\right)\left(1 - e^{-\beta\left|\hat{\mathbf{g}}(n)\right|}\right), \tag{6}$$

where β is a regulation control parameter for the LFA, and it is greater than zero. For this LFA function, its behavior has been investigated in [19] with different parameter β. When β takes a large value, the LFA will approximate to be a l_0-norm. It is similar with l_1-norm when a vary small β is chosen. Therefore, a proper selection for β will obtain a flexible LFA penalty. The derivation of above LFA penalty is

$$S'_\beta\left(\hat{\mathbf{g}}(n)\right) = (1 + \beta)\,e^{\left(-\beta\left|\hat{\mathbf{g}}(n)\right|\right)}\mathrm{sgn}\left(\hat{\mathbf{g}}(n)\right). \tag{7}$$

Then, we introduce the derivation of LFA penalty into the JO-NLMS algorithm to develop a fire-new zero attractor, and the updating equation of the LFAJO-NLMS is

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \frac{\left[m(n-1) + N\delta_n^2\right] e(n)\,\mathbf{x}(n)}{M\delta_r^2 + (M+2)\,\delta_x^2\left[m(n-1) + N\delta_n^2\right]} - \rho(1 + \beta)\,e^{\left(-\beta\left|\hat{\mathbf{g}}(n)\right|\right)}\mathrm{sgn}\left(\hat{\mathbf{g}}(n)\right), \tag{8}$$

where ρ is a controlling parameter of the proposed LFAJO-NLMS algorithm. The last term $-\rho(1+\beta)e^{(-\beta|\hat{\mathbf{g}}(n)|)}\mathrm{sgn}(\hat{\mathbf{g}}(n))$ is our proposed LFA zero attractor, which can give a high probability zero attracting on the zero or near zero coefficients. Therefore, the proposed LFAJO-NLMS algorithm can provide fast convergence and small steady-state error by using the proposed LFA zero attractor.