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Analytical study of lateral-circulation-induced exchange flow in tidally dominated well-mixed estuaries

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ABSTRACT

In straight estuary channels, differential advection and the Coriolis force are the major driving mechanisms for lateral circulation. An analytical model was developed to explore the roles of the two mechanisms in the dynamics of tidally dominated well-mixed estuaries. The model provided a nondimensional parameter, K_{eh}, a type of Kelvin number (considered as horizontal Kelvin number) to elucidate the relative importance of the two mechanisms. Differential advection is effective under small K_{eh}, while the Coriolis force is effective under larger K_{eh}. The critical value of K_{eh} has an order of magnitude of 0.1 in well-mixed estuaries. Lateral circulations generate residual currents through the lateral advection term in the along-estuary momentum equation. When differential advection is effective, the lateral-advection-induced flow has a laterally sheared structure with the landward flow in the channel and seaward flows over shoals. When the Coriolis force is effective, it has a laterally sheared structure with the landward flow in the left part of the cross-section and the seaward flow in the right (facing ocean). When the two mechanisms are equally important, it has an asymmetric laterally sheared structure with a stronger seaward flow over the right shoal. Those lateral structures indicate that the lateral-circulation-induced flow generally reinforces the estuarine gravitational circulation.

1. Introduction

It has long been assumed that the estuarine circulation (or exchange flow) is driven by the along-estuary density gradient, relying on a linear momentum balance between horizontal pressure gradient and friction (Pritchard, 1956). Increasingly recent studies have recognized that the nonlinear advection terms in the along-estuary momentum equation, in particular those related to lateral circulation, also play an important role in estuarine dynamics (Smith, 1976; West and Mangat, 1986; Scott, 1994; Seim and Gregg, 1997; Valle-Levinson et al., 2000; Lacy and Monismith, 2001; Chant, 2002). Lerczak and Geyer (2004) investigated the contribution of lateral advection to estuarine exchange flow using an idealized numerical model, and found that the vertical structure of tidally averaged lateral advection is of similar shape to the pressure gradient, and helps reinforce the two-layer estuarine circulation. Huijts et al. (2006, 2009, 2011) developed analytical models to investigate transverse residual flows in tidal estuaries and demonstrated that the interaction between tidal currents and lateral density gradients fundamentally changed the transverse structure of residual flows in many estuaries. Cheng and Valle-Levinson (2009) used a pair of nondimensional numbers (e.g. Rossby, U/fB and Ekman numbers, K_m/fH^2, where U is the velocity scale, B and H are the width and depth of the estuary, f is the Coriolis parameter, and K_m is the vertical eddy viscosity), to evaluate the relative importance of lateral advection and the Coriolis force in estuarine dynamics, and showed that lateral advection is most effective at large Rossby number and small Ekman number. However, their numerical experiments neglected tidal processes, and therefore the lateral advection was related to the subtidal lateral circulation. In tidal time scales, various lateral circulations can be developed due to different driving mechanisms, and typically exhibit distinct intratidal variability.

Because the redistribution of along-estuary momentum depends on the lateral circulation, it is necessary to understand the basic characteristics of lateral tidal circulation in order to reveal how lateral advection affects residual currents in estuaries. The development of lateral circulation in estuaries can be illustrated using the conservation equation of vorticity based on Chant (2010) and Li and Li (2012):

$$\frac{d\hat{\Omega}_x}{d\hat{t}} - (\hat{\boldsymbol{\Omega}}\cdot\nabla)\hat{u} = \hat{f}\frac{\partial \hat{u}}{\partial \hat{z}} - \frac{\hat{g}}{\hat{\rho}_0}\frac{\partial \hat{\rho}}{\partial \hat{y}} - \frac{2\hat{u}}{\hat{R}}\frac{\partial \hat{u}}{\partial \hat{z}} + (\nabla^2\cdot\hat{\boldsymbol{K}})\hat{\Omega}_x, \tag{1}$$

where the caret denotes dimensional variables, $\hat{u}$, $\hat{v}$ and $\hat{w}$ are the

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components of velocity vector corresponding to $\hat{x}$ (along-channel), $\hat{y}$ (cross-channel) and $\hat{z}$ (vertical) directions, $\hat{t}$ denotes time, $\hat{\mathbf{\Omega}}$ is the vorticity vector and $\hat{\Omega}_x$ ($=\frac{\partial \hat{w}}{\partial \hat{y}} - \frac{\partial \hat{v}}{\partial \hat{z}}$) is the component corresponding to $\hat{x}$ direction, $\hat{f}$ is Coriolis parameter, $\hat{g}$ is gravitational acceleration, $\hat{\rho}$ is water density, $\hat{\rho}_0$ is the reference water density, $\hat{R}$ is the radius of channel curvature, and $\hat{\boldsymbol{K}}$ is the eddy viscosity vector that has three components in $\hat{x}$, $\hat{y}$ and $\hat{z}$ directions respectively. On the left hand side of the equation, the first term is the time rate of change of $\hat{\Omega}_x$. The second term represents stretching and tilting of the lateral circulation (or vorticity) due to the nonuniform velocity field. On the right hand side of the equation, the first three terms represent driving mechanisms of lateral circulation showing the effects of the Coriolis force, lateral density gradient and channel curvature respectively, while the last term in the equation tends to dissipate the lateral circulation. The focus of this study is placed on straight estuary channels in which the channel curvature effect is negligible.

The Coriolis force is effective in coastal basins (Winant, 2007). It drives a one-cell lateral circulation in the transverse section through the vertical shear of the along-estuary flow such that the resultant lateral circulation is counterclockwise (facing ocean in the Northern Hemisphere) during the flood and is clockwise during the ebb. The lateral density gradient drives lateral circulation in a similar way that the along-estuary density gradient drives the estuarine gravitational circulation. The structure of lateral circulation depends on the direction of lateral density gradient that is not necessarily uniform across the section. The lateral density gradient arises from a variety of processes that can be illustrated by laterally differentiating the salt transport equation (Cheng et al., 2009; Chant, 2010):

$$\frac{\partial}{\partial \hat{t}}\frac{\partial \hat{s}}{\partial \hat{y}} = -\frac{\partial \hat{u}}{\partial \hat{y}}\frac{\partial \hat{s}}{\partial \hat{x}} - \frac{\partial \hat{v}}{\partial \hat{y}}\frac{\partial \hat{s}}{\partial \hat{y}} - \frac{\partial \hat{w}}{\partial \hat{y}}\frac{\partial \hat{s}}{\partial \hat{z}} + \frac{\partial}{\partial \hat{y}}\frac{\partial}{\partial \hat{z}}\left(\hat{K}_s\frac{\partial \hat{s}}{\partial \hat{z}}\right). \tag{2}$$

Here, $\hat{s}$ is salinity, $\hat{K}_s$ is the vertical eddy diffusivity. We only keep the terms that produce lateral density gradients, while the other terms resulted from the differentiation that only produce local changes in lateral density gradient are neglected. On the right hand side of the equation, the first term represents differential advection which generates lateral salinity gradients due to the advection of along-estuary salinity gradient by the lateral shear of along-estuary velocity (Nunes and Simpson, 1985). The second and third terms modify existing lateral salinity gradients by compressing or tilting isohalines, representing the contribution of lateral circulation. The fourth term represents lateral variations in mixing that might be called differential diffusion in stratified estuaries (Cheng et al., 2009). Nunes and Simpson (1985) revealed that differential advection is an important mechanism driving lateral circulation. They showed that during flood tides differential advection produces higher salinity at the thalweg than over shoals due to the faster currents, and thus drives a pair of counter-rotating lateral flow cells in the cross-section. The water converges over the deep channel and diverges near the bottom forming a downwelling in the middle of the cross-section. During ebb tides the direction of tidal currents reverses and differential advection is expected to result in a pair of lateral flow cells with a reversed pattern comparing to that during flood tides.

Differential advection and the Coriolis force drive two different types of lateral circulations (e.g. one-cell structure versus two-cell structure) which in turn affect the residual currents in different ways. Still missing are criteria to determine the relative importance of the two mechanisms in driving lateral circulation in estuaries, and the characteristics of the along-estuary residual circulation generated by the two types of lateral circulations through tidally averaged lateral advection. To reach the two specific objectives, this study is to develop an analytical model that is able to compare the two mechanisms in a united model. Also, a time varying lateral salinity gradient obtained by solving the lowest order salt transport equation is included in the model to show the effect of differential advection.

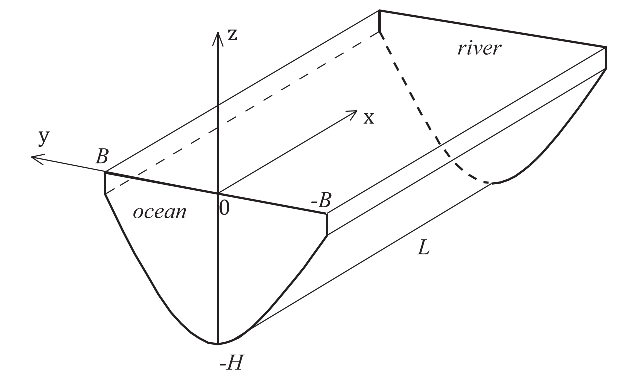

Fig. 1. Schematic of an idealized estuary channel with the local coordinates labeled. The model domain is a cross-section of the channel.

The remainder of this paper is structured as follows: Section 2 describes the development of the analytical model and proposes a nondimensional parameter to elucidate the relative importance of differential advection and the Coriolis force; Section 3 concentrates on the lowest order solution of the model that depicts lateral tidal circulation; Section 4 concentrates on the first order solution of the model that depicts the residual circulation, and Section 5 presents discussion and summarizes the main findings of this study.

2. The model

2.1. Governing equations

The model describes a cross-section in an elongated straight estuary channel with a varying cross-channel bottom bathymetry and a constant width, $2\hat{B}$, on the f plane (Fig. 1). Although the model is two-dimensional, it is necessary to consider a three-dimensional estuary channel in order to choose appropriate variables to scale the governing equations. The length $\hat{L}$ is much larger than the width, and the maximum depth, $\hat{H}$ is much smaller than $\hat{B}$. The origin of the local coordinate system is at the surface of the middle of the cross-section of the estuary. The $\hat{x}$ coordinate positively extends landward and the $\hat{y}$ coordinate is orthogonal to $\hat{x}$ varying from $-\hat{B}$ to $\hat{B}$. A semidiurnal tide (S_2) enters from the mouth of the estuary and freshwater discharge is imposed at the head of the estuary ($\hat{x} = \hat{L}$). The dimensional governing equations for the estuary include

$$\frac{\partial \hat{u}}{\partial \hat{t}} + \hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} + \hat{w}\frac{\partial \hat{u}}{\partial \hat{z}} - \hat{f}\,\hat{v} = -\hat{g}\frac{\partial \hat{\eta}}{\partial \hat{x}} + \frac{\hat{g}}{\hat{\rho}_0}\frac{\partial \hat{\rho}}{\partial \hat{x}}z + \frac{\partial}{\partial \hat{z}}\left(\hat{K}_v\frac{\partial \hat{u}}{\partial \hat{z}}\right), \tag{3a}$$

$$\frac{\partial \hat{v}}{\partial \hat{t}} + \hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} + \hat{w}\frac{\partial \hat{v}}{\partial \hat{z}} + \hat{f}\,\hat{u} = -\hat{g}\frac{\partial \hat{\eta}}{\partial \hat{y}} + \frac{\hat{g}}{\hat{\rho}_0}\frac{\partial \hat{\rho}}{\partial \hat{y}}z + \frac{\partial}{\partial \hat{z}}\left(\hat{K}_v\frac{\partial \hat{v}}{\partial \hat{z}}\right), \tag{3b}$$

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} + \frac{\partial \hat{w}}{\partial \hat{z}} = 0, \tag{3c}$$

$$\frac{\partial \hat{\eta}}{\partial \hat{t}} + \frac{\partial}{\partial \hat{x}}\left(\int_{-\hat{h}}^{\hat{\eta}} \hat{u}\,d\hat{z}\right) + \frac{\partial}{\partial \hat{y}}\left(\int_{-\hat{h}}^{\hat{\eta}} \hat{v}\,d\hat{z}\right) = 0, \tag{3d}$$

$$\frac{\partial \hat{s}}{\partial \hat{t}} + \hat{u}\frac{\partial \hat{s}}{\partial \hat{x}} = \frac{\partial}{\partial \hat{z}}\left(\hat{K}_v\frac{\partial \hat{s}}{\partial \hat{z}}\right), \tag{3e}$$

$$\hat{\rho} = \hat{\rho}_0(1+\hat{\beta}\hat{s}). \tag{3f}$$

Here $\hat{\beta}$ is haline contraction coefficient ($7.7\times10^{-4}\,\text{psu}^{-1}$), $\hat{K}_v$ is the vertical eddy viscosity, the horizontal salinity gradients are assumed independent of depth, and Schmidt number is assumed 1.0. For the salt transport, we neglected the horizontal diffusion and the redistribution of salt by lateral circulation. This treatment leaves differential advection the driving mechanism of salt transport (impacts of this treatment on model solution are given in the Section 5). The vertical boundary