



## Review

# Fisheries management in random environments: Comparison of harvesting policies for the logistic model



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## ABSTRACT

We describe the growth dynamics of a harvested fish population in a random environment using a stochastic differential equation logistic model, where the harvest term depends on a constant or a variable fishing effort. We consider revenues to be proportional to the yield and costs to be quadratic in terms of effort. We compare the optimal expected profit obtained with two types of harvesting policies, one based on variable effort, which is inapplicable, and the other based on a constant effort, which is applicable and sustainable. We answer two new questions: (a) What is the constant effort that optimizes the expected profit per unit time? (b) How do the two policies compare in terms of performance? We show that, in a realistic situation, there is only a slight reduction in profit when choosing the applicable constant effort policy instead of the inapplicable policy with variable effort.

## 1. Introduction

In a deterministic environment, the logistic growth model for a harvested population can be described, in terms of the *per capita* growth rate, by the ordinary differential equation (ODE)

$$\frac{1}{X(t)} \frac{dX(t)}{dt} = r \left( 1 - \frac{X(t)}{K} \right) - qE(t), \quad X(0) = x, \quad (1)$$

where  $X(t)$  is the population size at time  $t$ , measured as biomass or as number of individuals,  $r > 0$  is the intrinsic growth rate of the population,  $K > 0$  is the carrying capacity of the environment,  $q > 0$  is the catchability coefficient,  $E(t) \geq 0$  is the fishing effort and  $X(0) = x > 0$  represents the population size at time 0. The yield per unit time from harvesting is denoted by  $H(t) = qE(t)X(t)$ .

However, the environment is subject to significant random fluctuations that affect the population *per capita* natural growth rate. The effect of these fluctuations can be approximated by a white noise  $\sigma\epsilon(t)$ , where  $\epsilon(t)$  is a standard white noise and  $\sigma > 0$  measures the strength of environmental fluctuations. Therefore, the above ODE Eq. (1) must be updated to the stochastic differential equation (SDE)

$$\frac{1}{X(t)} \frac{dX(t)}{dt} = r \left( 1 - \frac{X(t)}{K} \right) + \sigma\epsilon(t) - qE(t), \quad X(0) = x,$$

which can be written in the standard format

$$dX(t) = rX(t) \left( 1 - \frac{X(t)}{K} \right) dt - qE(t)X(t)dt + \sigma X(t)dW(t), \quad X(0) = x, \quad (2)$$

where  $W(t) = \int_0^t \epsilon(s)ds$  is a standard Wiener process. We will assume that  $r > \sigma^2/2$ , otherwise the population will rendered extinct, even in the absence of harvesting (see Braumann, 1985).

Stochastic differential equations have been studied as a way to explain many physical, biological, economic and social phenomena. A particular case is the application (starting with the pioneering work of Beddington and May (1977)) to the growth dynamics of a harvested population subject to a randomly varying environment, with the purpose of obtaining optimal harvesting policies. Such policies usually are intended to maximize the expected yield or profit over a finite or infinite time horizon  $T$ . Since population size depends on the fishing effort, it seems natural to consider  $E(t)$  as a control and apply optimal control techniques to achieve either yield or profit optimization, discounted by a social rate.

The profit per unit time can be defined as the difference between sales revenue and fishing costs, i.e.,

$$P(t) := R(t) - C(t),$$

where  $R(t)$  and  $C(t)$  are respectively the revenue and cost per unit time. We consider the revenue per unit time to be proportional to the yield,

$$R(t) = pH(t),$$

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where  $p > 0$  is the price per unit of yield. The cost of harvest per unit time is assumed to depend on effort and to have a quadratic form given by

$$C(t) = c(E(t))E(t), \quad \text{with } c(E(t)) = c_1 + c_2E(t),$$

where  $c(E(t))$  is the cost per unit effort and  $c_1, c_2 > 0$  are constants. The quadratic cost structure incorporates the case where the fishermen need to use less efficient vessels and fishing technologies or pay higher overtime wages to implement an extraordinary high effort (see Clark, 1976, 1990). However, other more complicated profit structures can be used, as well as other population growth models (for instance, the Gompertz model instead of the logistic). The methodology would be similar to the one we use in this paper.

In the deterministic case, there is a quite comprehensive account of optimal harvesting policies regarding yield or profit optimization (Clark, 1976, 1990). Under general assumptions, unless we are close to the end of a finite time horizon  $T$ , the optimal policy is to harvest with maximum intensity (which can be limited to a maximum harvesting effort or be unlimited) when the population is above a critical threshold and stop harvesting (zero effort) when the population is below that threshold. Once the threshold is reached, one just needs to keep the harvesting rate constant at an appropriate value so that the population remains at the threshold size. However, when the population is below the threshold, the fishery should be closed until the threshold is reached, which may take a while.

Stochastic optimal control methods were also applied to derive optimal harvesting strategies in a randomly varying environment (e.g. Alvarez, 2000a,b; Alvarez and Shepp, 1998; Arnason et al., 2004; Hanson and Ryan, 1998; Lande et al., 1994, 1995; Lungu and Øksendal, 1996; Suri, 2008). The optimal policy is similar to the deterministic case, i.e., harvest with maximum intensity when the population is above a critical threshold (not necessarily the same as in the deterministic case) and stop harvesting when below the threshold. However, after the threshold size is attained, due to random fluctuations of the environment, population size will keep varying. In this case, fishing effort must be adjusted at every instant, so that the size of the population does not go above the equilibrium value. Such policies imply that the effort changes frequently and abruptly, according to the random fluctuations of the population. Sudden frequent transitions between quite variable effort levels are not compatible with the logistics of fisheries. Besides, the period of low or no harvesting poses social and economical undesirable implications. In addition to such shortcomings, these optimal policies require the knowledge of the population size at every instant, to define the appropriate level of effort. The estimation of the population size is a difficult, costly, time consuming and inaccurate task and, for these reasons, and the others pointed above, these policies should be considered unacceptable and inapplicable.

In Braumann, 1981, 1985, 2008, a constant fishing effort,  $E(t) \equiv E$ , was assumed, providing an alternative approach to optimal harvesting. For a large class of models (including the logistic), it was found that, taking a constant effort in Eq. (2), there is, under mild conditions, a stochastic sustainable behaviour. Namely, the probability distribution of the population size at time  $t$  will converge, as  $t \rightarrow +\infty$ , to an equilibrium probability distribution (the so-called stationary or steady-state distribution) having a probability density function (the so-called stationary density). For the logistic model, the stationary density function was found, and the effort  $E$  that optimizes the steady-state yield was determined. The issue of profit optimization, however, was not addressed.

This paper considers this issue of profit optimization for the sustainable constant effort harvesting policy. This policy, rather than switching between large and small or null fishing effort, keeps a constant effort and is therefore compatible with the logistics of fisheries. Furthermore, this alternative policy does not require knowledge of the population size. However, it will result in a reduction of the profit when we compare it with the inapplicable optimal policy. We will examine if

such reduction is appreciable or negligible.

Section 2 presents the approach to solve the optimization variable effort problem through a dynamic programming method. In Section 3 we present the alternative sustainable approach based on constant effort. Section 4 shows an application with realistic biological and fishing parameters in which the two policies are compared using numerical and Monte Carlo methods. We end up, in Section 5, with the conclusions.

Computations were carried out with R (<http://r-project.org>) and the code is available as supplementary material.

## 2. Variable effort optimal policy

This section will summarize the variable effort optimal policy under a randomly varying environment. We will start the optimization at time  $t = 0$ . Let  $X(0) = x$  be the corresponding population size. Furthermore, harvesting continues up to the time horizon  $T < +\infty$  and we work with the profit present value, i.e., future profits are discounted by a rate  $\delta > 0$  accounting for interest rate and cost of opportunity losses and for other social rates. For a time  $t$  in the horizon  $[0, T]$ , we define

$$J(y, t) := \mathbb{E} \left[ \int_t^T e^{-\delta(\tau-t)} P(\tau) d\tau | X(t) = y \right], \tag{3}$$

which is, at time  $t$ , the expected discounted future profits when the population size at that time is  $y$ .

We want to optimize the expected accumulated discounted profit earned by the harvester in the interval  $[0, T]$ ,

$$\begin{aligned} V := J(x, 0) &= \mathbb{E}_x \left[ \int_0^T e^{-\delta\tau} P(\tau) d\tau \right] \\ &= \mathbb{E}_x \left[ \int_0^T e^{-\delta\tau} (pqX(\tau) - c_1 - c_2E(\tau))E(\tau) d\tau \right], \end{aligned}$$

where we denote  $\mathbb{E}[\dots | X(0) = x]$  by  $\mathbb{E}_x[\dots]$ .

Given that  $E(t)$  is used as a control, the optimization is carried out with respect to  $E(t)$ . A very important issue emerges when dealing with fishing effort: should one consider any constraints on effort? In practice, the effort is always non-negative, hence we must consider  $E(t) \geq 0$ . On the other hand, the number of tools, gears, hours, vessels and manpower is finite and limited, so we will consider effort to be constrained as

$$0 \leq E(t) \leq E_{\max} < \infty. \tag{4}$$

The optimization problem can be solved by stochastic dynamic programming theory through Bellman's principle of optimality (see Bellman, 1957). In terms of optimization theory, our problem is to find the effort that maximizes  $V$ , subject to the growth dynamics given by Eq. (2) and to the constrains on effort given by Eq. (4). In addition, from Eq. (3) we get  $J(X(T), T) = 0$ , which is a boundary condition. Summing up, the stochastic optimal control problem is to determine

$$V^* := J^*(x, 0) = \max_{\substack{E(\tau) \\ 0 \leq \tau \leq T}} \mathbb{E}_x \left[ \int_0^T e^{-\delta\tau} (pqX(\tau) - c_1 - c_2E(\tau))E(\tau) d\tau \right], \tag{5}$$

s.t.

$$dX(t) = rX(t) \left( 1 - \frac{X(t)}{K} \right) dt - qE(t)X(t)dt + \sigma X(t)dW(t), \quad X(0) = x,$$

$$0 \leq E(t) \leq E_{\max} < \infty,$$

$$J^*(X(T), T) = 0.$$

The maximizer, i.e., the effort function  $E(t)$  that leads to the maximum  $V^*$ , will be called the optimal variable effort and will be denoted by  $E^*(t)$ .

To solve Eq. (5), one can employ stochastic dynamic programming to derive the Hamilton-Jacobi-Bellman (HJB) equation (see Hanson, 2007)

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