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Attribution of horizontal and vertical contributions to spurious mixing in an Arbitrary Lagrangian–Eulerian ocean model

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ABSTRACT

We examine the separate contributions to spurious mixing from horizontal and vertical processes in an ALE ocean model, MOM6, using reference potential energy (RPE). The RPE is a global diagnostic which changes only due to mixing between density classes. We extend this diagnostic to a sub-timestep timescale in order to individually separate contributions to spurious mixing through horizontal (tracer advection) and vertical (regridding/remapping) processes within the model. We both evaluate the overall spurious mixing in MOM6 against previously published output from other models (MOM5, MITGCM and MPAS-O), and investigate impacts on the components of spurious mixing in MOM6 across a suite of test cases: a lock exchange, internal wave propagation, and a baroclinically-unstable eddying channel.

The split RPE diagnostic demonstrates that the spurious mixing in a lock exchange test case is dominated by horizontal tracer advection, due to the spatial variability in the velocity field. In contrast, the vertical component of spurious mixing dominates in an internal waves test case. MOM6 performs well in this test case owing to its quasi-Lagrangian implementation of ALE. Finally, the effects of model resolution are examined in a baroclinic eddies test case. In particular, the vertical component of spurious mixing dominates as horizontal resolution increases, an important consideration as global models evolve towards higher horizontal resolutions.

1. Introduction

One of the myriad uses of ocean models is in developing ocean heat uptake estimates and overturning circulation predictions (Armour et al., 2016). Additionally, the overturning circulation itself affects the wider climate, which manifests when ocean models are used as a component of coupled climate simulations. The strength of ocean heat uptake and the overturning circulation are both strongly controlled by the density structure of the ocean, which is modified by mixing. For example, mixing at depth modifies the abyssal overturning cell that constitutes part of the meridional overturning circulation (Mashayek et al., 2015), while the time scale of adjustment of the overturning circulation toward equilibrium is sensitive to near-surface mixing (Vreugdenhil et al., 2015). A consequence of this sensitivity is that ocean models with significant mixing due to numerical truncation errors (spurious mixing) are unlikely to accurately constrain the abyssal overturning.

Numerical ocean models are governed by approximations of the incompressible Navier–Stokes equations for momentum, also known as the primitive equations (Griffies, 2004). In these models, the vertical balance is hydrostatic, where the vertical pressure gradient force is matched by the gravitational force. The mixing of momentum by the unresolved eddy field from the mesoscale down to the Kolmogorov scale is parameterised by an explicit eddy viscosity term. Potential density of water parcels is a function of salinity and potential temperature through an equation of state. These tracers are advected by the explicitly resolved eddy field, and mixed by the unresolved eddy field through a parameterised eddy diffusivity term. Due to the vast difference between lateral and vertical scales in models, the eddy viscosity and diffusivity terms are often separated into lateral and vertical components.

To solve the primitive equations, ocean models implement some kind of discretisation, such as the finite volume method. This discretisation involves representing the computational domain as a series

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of grid cells in three-dimensional space, where each grid cell has associated mean velocities and tracer concentrations, and possibly higher moments (Prather, 1986). Horizontal tracer advection schemes are discretisations of the advection equation that use information from neighbouring grid cells to create higher-order reconstructions of the tracer field than that which is stored directly in the cell. Mixing processes create fluxes of tracer between grid cells. In ocean models, mixing has two main causes, physical and numerical. The physical mixing comes from advection by numerically unresolved turbulence, which is typically parameterised as a diffusive process. On the other hand, numerical mixing arises from truncation errors in the discretisations and algorithms used by the ocean model to solve the governing equations. Numerical mixing is also known as spurious mixing and has no physical basis. For example, first-order upwind advection has numerical diffusion as the leading error term (Gentry et al., 1966).

Spurious mixing is undesirable in ocean models as it is unphysical and may add to the imposed and parameterised mixing to an unknown extent. Spurious mixing affects numerical experiments which are contingent on the density structure of the ocean. Ocean heat uptake or overturning circulation strength in such experiments may be biased (Griffies et al., 2015). One of the considerations in model development and configuration is thus to ensure spurious mixing is minimised.

The magnitude of spurious mixing is strongly influenced by the choice of horizontal tracer advection scheme. Much of the focus in reducing spurious mixing has therefore been on tracer advection, through improving numerical accuracy or the model's tracer sub-gridscale representations. Some argue that a high-order advection scheme is sufficient to reduce the spurious mixing to acceptable levels (Daru and Tenaud, 2004). This is simply a matter of using a sufficiently high-order polynomial reconstruction to try to capture the overall structure of tracer distributions. Other advection schemes attempt to preserve the sub-gridscale representation of a given field. For example, by carrying information about both first and second-order moments, the Prather (1986) method is able to reconstruct a field to second order. This second-order moment scheme must often be used in conjunction with a flux limiter to avoid the creation of spurious minima and maxima; these limiters in effect lead to a sub-cell diffusion (Morales Maqueda and Holloway, 2006). An alternative view is that the tracer advection scheme only needs sufficient accuracy before grid-scale noise in velocity becomes the dominant source of spurious mixing (Ilıcak et al., 2012).

A second consideration in model configuration in order to minimise spurious mixing is the vertical coordinate. We first describe some of the main choices for the vertical coordinate in ocean models. The basis for the z-family of coordinates is the pure z-level coordinate, where coordinate surfaces are simply fixed geopotentials. The first extension to the z-level coordinate is z^* or z-star, which individually and uniformly expands or contracts water columns to accommodate changes in the free surface height (Adcroft and Campin, 2004). z-family coordinates allow for ahead-of-time specification of vertical resolution, which must be applicable to the entire modelled domain and thus sufficiently general. Some disadvantages of these coordinates are poor representation of overflows (Legg et al., 2009), and the spurious diapycnal mixing associated with purely horizontal coordinate surfaces, e.g. isoneutral diffusion (Griffies et al., 2000). Instead of being referenced to physical positions, isopycnal coordinate models use potential density as the vertical coordinate. This formulation completely eliminates spurious diapycnal mixing, as well as providing enhanced vertical resolution at sharp density fronts. However, there are difficulties in representing the nonlinear equation of state, as there is no conservative density coordinate that is monotonic with depth (Griffies, 2004). Additionally, the surface mixed layer is essentially unstratified, and hence is a region of very low vertical resolution.

Hybrid vertical coordinates combine or modify other vertical coordinates to optimise their performance, at the expense of complexity and computational cost. One hybrid vertical coordinate is z-tilde (Leclair and Madec, 2011), which has Lagrangian behaviour (i.e., the grid is advected by the vertical velocity) for motions on short timescales, but relaxes to a target z-star grid over long timescales to prevent the grid from drifting. This scheme was demonstrated to reduce spurious mixing when modelling the propagation of internal gravity waves. A final example is the continuous isopycnal coordinate (White et al., 2009), where instead of layers having a predefined density as in the pure isopycnal coordinate, interfaces have a target density. In this case, there must be dynamic adjustment of the coordinate surfaces in order to maintain the target density. The release of the constraint to layered isopycnals means that further physical processes can be more easily added to the model, such as geothermal heating or double diffusion (White et al., 2009). In isolation, each coordinate has strengths and weaknesses for ocean modelling, but the combination attempts to preserve the strengths of each.

To allow generalised vertical coordinates, models can make use of an Arbitrary Lagrangian–Eulerian (ALE) scheme. There are two general implementations of ALE in ocean models, depending on the reference frame of the model (Margolin and Shashkov, 2003; Leclair and Madec, 2011). In quasi-Eulerian models, any changes in the vertical grid due to the choice of coordinate are incorporated into the solution of the primitive equations (Kasahara, 1974). Incorporating changes in the vertical grid is often done by calculating the motion of the new vertical grid relative to the old grid as a vertical velocity. As such, there could be an associated spurious mixing with advection in both the horizontal and vertical directions.

The quasi-Lagrangian algorithm (Hirt et al., 1974; Bleck, 2002) is for models which are primarily implemented in a Lagrangian frame of reference (such as MOM6, which is the focus of this paper; see Jansen et al., 2015). Here, the vertical grid may adjust during the dynamic solution of the primitive equations or as a consequence of parameterisations such as Gent-McWilliams thickness diffusion (Gent and McWilliams, 1990). This dynamical timestep incorporates most of the modelled processes, including the calculation and application of advective tracer fluxes. Typically, these fluxes are accumulated during the solution of the primitive equations and applied after the primitive equations are solved, on the updated grid. The model dynamics are then followed by the ALE timestep, which consists of two phases. In the first phase, referred to here as *regridding*, a new vertical grid is calculated using the current model state. This new grid may be as simple as a prescribed z-star coordinate, or could be a function of local density or depth. Secondly, the new grid is applied in the *remapping* phase, during which the model state is mapped onto the new grid. The remapping algorithm is often an adaptation of an advection scheme (Margolin and Shashkov, 2003), although other conservative algorithms may be used. Remapping differs from vertical advection in that the effective vertical velocities have a non-physical component to recover the new grid, as well as a physical component. Spurious mixing that occurs during the remapping phase therefore depends on the vertical dynamics, the new grid and the sub-gridscale reconstruction of tracers on the old grid.

The accuracy of the reconstruction scheme used in the remapping stage of ALE was investigated by White and Adcroft (2008) with their piecewise quartic method (PQM). PQM is the most accurate reconstruction method available in MOM6, and was found to significantly increase reconstruction accuracy for a small increase in computational cost compared to limited PPM (piecewise parabolic method). The impacts of different reconstruction schemes in regridding and remapping were considered by White et al. (2009), comparing their spurious mixing in terms of the change of volume distributions across density classes. Neither of these studies quantified the magnitude of spurious mixing in total, or as a comparison to the spurious mixing by horizontal advection. Formulating this comparison is one of the aims of this paper.

There is no consensus on the appropriate diagnostic technique to use to evaluate the performance of numerical schemes with regard to spurious mixing. Griffies et al. (2000) used an effective diapycnal