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Scale–location specific soil spatial variability: A comparison of continuous wavelet transform and Hilbert–Huang transform

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ABSTRACT

Soil spatial variability has become the rule rather than the exception; it is the consequence of spatial dependence, periodicity, nonstationarity, and nonlinearity. The continuous wavelet transform (CWT) has been extremely useful in revealing scale- and location-specific information of nonstationary soil spatial variation. The Hilbert–Huang transform (HHT) has also been used in soil science to reveal scales and locations of variations in soil properties. These variations may be controlled by the underlying soil processes that can also be represented using a linear or nonlinear equation/function. The objective of this manuscript was to compare the inherent strengths and weaknesses of CWT and HHT in quantifying scale- and location-specific soil spatial variation. Examples using four simulated spatial series (stationary–linear, stationary–nonlinear, nonstationary–linear, and nonstationary–nonlinear) and two real world measurements of soil properties (organic carbon and soil water storage) were used to compare the methods. With its algorithmic basis, HHT identified the scale components present in the spatial series more flexibly, while the redundancy in CWT identified a diffuse band of scales as it is based on an underlying mathematical model. Additionally, the CWT identified variations that were biased towards large scales. The HHT used a more flexible basis for interpreting real data and could deal with nonlinear issues, while CWT could not. A similar result was also observed for soil organic carbon and soil water storage. Both methods could produce certain levels of information but the choice should be made based on the type of information that is required while taking into consideration the underlying assumptions. For example, to quantify the scale- and location-specific spatial variability of soil properties as controlled by soil processes which can be represented by a nonlinear equation, one achieves benefits from using HHT rather than CWT. In this case study, HHT showed superior performance in identifying scales and locations of soil spatial variability over CWT. In this study, HHT is compared with CWT only and needs further comparison with other types of wavelet analysis.

1. Introduction

Soil spatial variability has become the rule, not the exception, and is generally a product of the combined effect of soil physical, chemical, and biological processes that operate in different intensities and at different scales (Goovaerts 1998). Adequate understanding of soil variability as a function of space and scale is necessary for environmental prediction, precision agriculture, soil quality assessment, and natural resource management (Trangmar et al. 1985; Goderya 1998). A detailed description of soil spatial variability also provides critical information for the development of various logical, empirical, and physical models of soil landscape processes (Corwin et al. 2006).

Since the classic study of Nielsen et al. (1973), the systematic study of soil spatial variability has identified various complex issues, such as spatial dependence, periodicity, nonstationarity, and nonlinearity. While the similarity between two points can be dependent on the separation distance (spatial dependence), similarity can vary in a cyclical pattern (periodicity) or exhibit long increasing/decreasing trends (nonstationarity). Similarly, soil spatial variation controlled by processes can also be represented by a linear/nonlinear function/equation (nonlinearity) (Biswas and Si 2011a). Geostatistical analysis can measure the spatial dependence based on autocorrelation (Trangmar et al. 1985). In spectral analysis, a spatial series with periodicity is compared with the cyclic Fourier series (e.g. sine or cosine wave) and the characteristic period (= 1/frequency) of the series is used to represent the scale of underlying processes creating the periodic variability (Brillinger 2001). These analyses are based on a subtle assumption of stationarity in the spatial data that means the statistical properties of the variable under consideration depend on their relative locations. Thus, these analyses lose spatial information during the conversion to scale information. Often, soil spatial variations exhibit nonstationary trends in the spatial data arising from smooth and predictable linear

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and non-linear trends. Additionally, soil processes operating at different scales may be localized in space relative to the entire spatial domain (Si 2003). The nonstationarity and scale-localization restrict the use of geostatistics and spectral analysis for quantifying soil spatial variation (Oliver and Webster 1991; Goderya 1998).

Wavelet analysis has been used for almost two decades to study multi–scale (Bosch et al. 2004) soil spatial variability exhibiting stationary/nonstationary characteristics (Lark and Webster 1999; Lark et al. 2004; Si and Farrell 2004; Neupauer and Powell 2005; Biswas et al. 2008; Biswas and Si 2011c; 2011a; Biswas et al. 2013b; Biswas 2014; She et al. 2016; Tarquis et al. 2017). It partitions the total variation into positions (or locations) and frequencies (or scales) and thus, can deal with nonstationarity trends and scale-localization. There are several types of wavelet analyses, such as continuous wavelet transform (CWT), discrete wavelet transform (DWT), maximum overlap discrete wavelet transform (MODWT) and wavelet packet transform (WPT). These are a suite of tools used for different purposes, each with inherent advantages and disadvantages (Graps 1995; Percival and Walden 2000; Biswas and Si 2011a; Zhang et al. 2016). CWT is particularly suited for scale analysis as it partitions the overall variations in a spatial series into continuous scales and locations and has been widely used in soil science (Lau and Weng 1995; Si and Zeleke 2005). A comprehensive review on the applications of CWT in soil science can be found in Biswas and Si (2011a).

While CWT has proven to be extremely useful in revealing the underlying variability of any soil properties, it embodies the assumption that the spatial series as the realization of the underlying processes can then be represented by linear equations (Huang et al. 1998). In a linear system, generally the overall response is equal to the linear sum of individual responses or processes. In complex natural systems, however, the total effect from multiple processes may be non–additive due to the interactive and complementary positive or negative effects. In this situation, the principle of superposition does not apply and the overall system is known to be nonlinear (Yan and Gao 2007; Biswas et al. 2013b). The Hilbert-Huang transform (HHT) is known to characterize time-frequency variations in a series that exhibits both nonstationarity and nonlinearity (Huang et al. 1998) and thus, has been extended to explore the scale–location specific information of a spatial series in soil science (Biswas et al. 2009; Biswas and Si 2011d; Zhou et al. 2016). The main advantage of HHT is that it can identify the hidden physical trends directly from data without imposing any mathematical rules (unlike mother wavelet in CWT) in the analysis. Unlike other methods, HHT does not have any a priori basis; rather it is intuitive, direct, adaptive and completely data driven (Huang et al. 1998). It not only extracts information on scale-localization and large trends from a nonstationary series, it identifies instantaneous scales from instantaneous frequency, a local frequency and deal with nonlinearity (Huang et al. 1998; Kijewski-Correa and Kareem 2006). CWT has also been tested to calculate instantaneous scales (Kijewski-Correa and Kareem 2006; Biswas et al. 2013b) using a modified Morlet wavelet. However, due to the mathematical complexity and the unavailability of a common computer program, CWT using the regular Morlet wavelet is still the most widely used method for identifying scale- and location-specific variations in soil science. While CWT has been widely used in soil science to quantify scale-location variability, HHT is increasing in use. However, before it can be widely accepted, the method needs a comprehensive comparison with the currently available and widely-used methods, such as CWT. Therefore, the main objective of this study was to compare the relative merits of CWT and HHT in revealing scale–location specific variations.

In comparing the methods, the manuscript was divided into three parts. Part one provided a brief description of both methods while part two compared both methods using several types of artificial spatial series. Finally, two examples of soil properties (soil organic carbon and soil water storage) were used to compare the methods in identifying scale–location specific variations. The methods were compared based on their ability to accurately identify dominant scales; to determine if there was any bias towards identifying a scale (e.g. large scale, small scale); to assess their ability to deal with nonstationary and nonlinear spatial series, and the physical meaning (or significance) of the identified scales.

2. Methodology

2.1. Continuous wavelet transform (CWT)

The CWT decomposes the overall variations in a spatial series into distinct locations (sample positions) as a function of continuous scales. Different wavelet functions, such as Haar, Mexican Hat, and Morlet can be used to calculate wavelet coefficients. These functions are called the mother wavelet function that can be stretched or contracted in space (x) and at different scales (s). The detailed theoretical description of CWT is well established in the literature and is beyond the scope of this paper. For the readers' interest, full descriptions can be found in Farge (1992) and Kumar and Foufoula-Georgiou (1997) among many others. Briefly, CWT can be defined as the convolution of a spatial series Y_i of length N ($i = 1, 2, \cdots, N$) along a transect with equal increments of distance δx (Torrence and Compo 1998),

$$W_i^Y(s) = \sqrt{\frac{\delta x}{s}} \sum_{j=1}^{N} Y_j \psi\left[(j-i)\frac{\delta x}{s}\right] \tag{1}$$

which can be implemented through a series of Fast Fourier Transform (FFT). The function ψ[]is the mother wavelet function or 'basic wavelet'. The parameter s is the dilation–contraction factor and it is associated with scales. It is often determined as fractional powers of two; $s_j = s_0 2^{j\delta j}$, $j = 0, 1, \ldots, J$ where s_0 is the smallest resolvable scale and J determines the largest scale, $J = \delta j^{-1} \log_2 (N\delta x/s_0)$. Wavelet coefficients, $W_i^Y(s)$ are expressed as $a + ib.$, where a and b are the real and imaginary components of $W_i^Y(s)$, respectively. The energy (strength of variation) associated with each scale and location can be calculated from the magnitude of wavelet coefficients (Qi and Neupauer 2008). Like the Fourier power spectrum, the wavelet power spectrum can be calculated as $|W_i^Y(s)|^2$. These wavelet power spectra are a function of scales and locations. Therefore, a better visualization of power spectra can be given by a contour plot with locations on the horizontal axis and scales on the vertical axis. The wavelet spectrum at a location and scale represents the local variance, and the sum of all local spectra is equal to the total variance. Therefore, it can be used to examine the scale–location specific variations in soil properties.

2.2. Hilbert–Huang transform (HHT)

The HHT is a two–step method. The first step, empirical mode decomposition (EMD), separates the variations present in a spatial series according to their characteristic scales. It decomposes the overall spatial pattern into a finite and often small number of intrinsic modes that are known as intrinsic mode functions (IMFs). Each IMF represents a characteristic scale of variability. In the second step, the Hilbert–spectral analysis (HSA), the instantaneous scales are calculated after applying the Hilbert transform to each IMF. The energy is calculated from the instantaneous amplitude, a product of Hilbert transform and is a function of scale and location. Detailed theory on HHT can be found in Huang et al. (1998) and Huang and Wu (2008).

Briefly, IMFs are extracted through a sifting process (Fig. 1). For a spatial series, $Y(x)$, one can identify the local maxima and local minima (Fig. 2). The maxima and minima are then joined by a cubic spline line to create the upper (UE) and lower (LE) envelope (Fig. 2). The mean value of the envelopes $m_1 = (UE + LE)/2$can be calculated locally and subtracted from the original spatial series to get the first prototype $h_1(x) = Y(x) - m_1(x)$, which is a function of space. This prototype will be an IMF provided it has satisfied the following conditions: 1) the