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Evaluation of analytic solutions for steady interface flow where the aquifer extends below the sea

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ABSTRACT

A computational approach is presented for steady Dupuit interface flow where the aquifer extends below the sea. A detailed approach is outlined to determine the head at the coastline so that the solution below the leaky seabed may be combined with any type of steady Dupuit interface flow in the aquifer below the land. The method allows for any inland boundary condition including specified head and specified flux; cases of freshwater lenses caused by infiltration are also considered. The approach is implemented in a Python script and a Jupyter Notebook.

1. Introduction

This Technical Note concerns analytic solutions for one-dimensional steady Dupuit interface flow in coastal aquifers where the aquifer extends below the sea. The sea is separated from the aquifer by a leaky seabed. A variety of solutions have been published for steady interface flow where the aquifer extends below the sea (e.g., Edelman, 1972; Bruggeman, 1999; Kooi and Groen, 2001; Morgan et al., 2015). Sikkema and van Dam (1982) provided a detailed mathematical treatment, which was used by Bakker (2006) to derive a complete set of analytic solutions for the case where flow in the aquifer below the land is confined and uniform. Evaluation of the solution by Bakker (2006) is complicated. It requires determination of the type of flow (four types are distinguished), and when the tip of the interface reaches the end of the seabed, the solution requires evaluation of elliptic integrals and an iterative approach to determine parameters.

This Technical Note is, in part, a response to the recent calls for reproducibility in computational hydrology (Fienen and Bakker, 2016; Hutton et al., 2016; Barba, 2016), where a case is made that computational results cannot be reproduced or scrutinized when the source code is not available. Here, a cookbook recipe is provided for the evaluation of the part of the solution of Bakker (2006) in the aquifer below the sea. The solution below the sea can be coupled to any type of flow in the aquifer below the land, which may be simulated with, e.g., the Strack potential (Strack, 1976). The recipe is implemented in a Python computer program and combined with several options for the boundary conditions in the aquifer below the land. A Jupyter Notebook is developed to evaluate the position of the interface for a variety of cases. A Jupyter Notebook is an interactive document that integrates text, computer code, and results (Kluyver et al., 2016). The Python code and Jupyter Notebook are available from Bakker (2017).

2. Solution below the sea bottom

Consider one-dimensional steady Dupuit interface flow in a vertical cross-section (Fig. 1). The aquifer extends below the sea and the saltwater is at rest. The depth of the interface may be obtained from the head in the aquifer with the Ghijben–Herzberg equation.

Below the sea, the aquifer is bounded on top by a leaky layer separating the sea from the aquifer, so that flow is semi-confined. In cases where the leaky seabed is absent, the leaky layer represents the vertical resistance to flow of the aquifer (Anderson, 2005; Bakker, 2014). The leakage through the leaky layer is approximated as vertical and computed as

$$q_z = \frac{h - h_s}{c} \tag{1}$$

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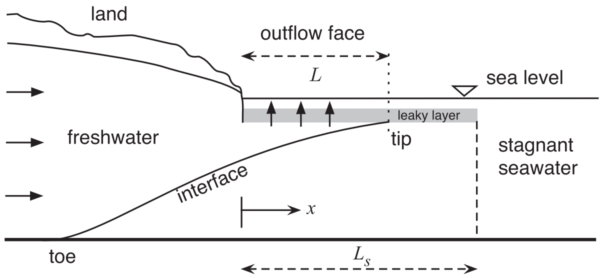

Fig. 1. Schematic cross-section of interface flow in an aquifer that extends below the sea. This example shows unconfined flow in the aquifer below the land.

where q_z [L/T] is the vertical component of the specific discharge vector through the leaky layer, h is the freshwater head in the aquifer, h_s is the freshwater head equivalent to the hydrostatic pressure in the saltwater at the top of the aquifer, and c [T] is the resistance to vertical flow of the leaky layer. The resistance c is computed from the thickness D and vertical hydraulic conductivity k_v of the leaky seabed as $c = D/k_v$. In absence of a physical leaky layer, the resistance c represents the resistance to vertical flow of the aquifer (Bakker, 2014). The leaky layer may have a finite length L_s or an infinite length. The hydraulic conductivity of the aquifer is k [L/T] and the thickness is H. The leakage factor λ [L] is defined as

$$\lambda = \sqrt{kHc} \tag{2}$$

The dimensionless density difference ν_s is defined as

$$\nu_s = \frac{\rho_s - \rho_f}{\rho_f} \tag{3}$$

where ρ_f and ρ_s are the densities of freshwater and saltwater, respectively. The main parameters of the problem are summarized in Table 1.

The flow in the aquifer below the land is not specified at this point. The discharge crossing the shoreline is called Q_0 [L^2/T], but is often unknown prior to solving the problem. Separate solutions are used for flow in the aquifer below the sea and for flow in the aquifer below the land. First, solutions are presented for flow below the sea, which result in equations for the head in the aquifer at the shoreline in terms of Q_0. A procedure to determine Q_0 from onshore boundary conditions is presented in a separate section. The shoreline is located at $x = 0$ (Fig. 1).

Equations are presented in terms of dimensionless variables. The dimensionless head ϕ is defined as

$$\phi = \frac{h - h_s}{\nu_s H} \tag{4}$$

The dimensionless head as a function of the dimensionless coordinate x/λ is governed by two dimensionless parameters, L_s/λ and μ, where the latter is defined as

$$\mu = \frac{Q_0 \lambda}{kH^2 \nu_s} \tag{5}$$

Table 1
Main parameters of the problem.

Symbol	Parameter	Dimensions
k	Hydraulic conductivity	L/T
H	Aquifer thickness	L
ρ_f	Density of freshwater	M/L^3
ρ_s	Density of saltwater	M/L^3
c	Resistance to vertical flow of leaky seabed	T
L_s	Length of leaky seabed	L
h_s	Sea level	L

Note that dimensionless parameter μ is a combination of the discharge Q_0 crossing the shoreline, the aquifer parameters, and the dimensionless density difference ν_s.

Four different types of flow are distinguished depending on the position of the tip and the toe of the interface. The tip of the interface is the location where the interface touches the top of the aquifer, while the toe of the interface is the location where the interface touches the bottom of the aquifer (Fig. 1). For type I, the toe of the interface is in the aquifer below the land and the tip of the interface does not reach the end of the semi-confined layer (Fig. 2a). For type II, the toe of the interface is in the aquifer below the sea and the tip of the interface does not reach the end of the semi-confined layer (Fig. 2b). For type III, the toe of the interface is in the aquifer below the land, and the tip of the interface is at the end of the semi-confined layer (Fig. 2c). For type IV, the toe of the interface is in the aquifer below the sea and the tip of the interface is at the end of the semi-confined layer (Fig. 2d).

The type of flow is a function of L_s/λ and the dimensionless parameter μ (Eq. (5)), which includes the discharge Q_0 crossing the shoreline. For example, when flow is of type I (Fig. 2a) and the discharge Q_0 increases, the toe of the interface moves towards the shoreline. If Q_0 is large enough, the toe will cross the shoreline (type II flow). The limiting case for which the toe is exactly at the shoreline is reached when $\mu = \sqrt{2/3}$, as derived by Bakker (2006).

In the following, a cookbook recipe is presented to determine the type of flow. An outline of the cookbook recipe is given in Fig. 3. Equations are given for the dimensionless head ϕ_0 at the shoreline and the length L of the outflow face for the different flow types. All equations are taken from Bakker (2006), where a detailed derivation is given. Following the recipe (Fig. 3), the first step is to compute the dimensionless parameter μ. The flow is of type I if $\mu < \sqrt{2/3}$ (and the length of the semi-confining layer is long enough, which will be checked later), and the dimensionless head ϕ_0 at the shoreline can be computed as

$$\phi_0 = \left(\frac{3\mu^2}{2}\right)^{1/3} \tag{6}$$

The length of the outflow face L is

$$L = (18\mu)^{1/3}\lambda \tag{7}$$

The flow is of type II if $\mu \geqslant \sqrt{2/3}$ (and the length of the semi-confining layer is long enough), and ϕ_0 and L can be computed as

$$\phi_0 = \frac{1 - \sqrt{2/3}}{2}\exp(-d/\lambda) + \frac{1 + \sqrt{2/3}}{2}\exp(d/\lambda) \tag{8}$$

$$L = \lambda\sqrt{6} + d \tag{9}$$

where

$$d = \lambda \ln\left[\frac{\mu + \sqrt{\mu^2 + 1/3}}{1 + \sqrt{2/3}}\right] \tag{10}$$

Next, it is checked whether the length of the semi-confining layer at the bottom of the sea is longer than the computed length of the outflow face L. If it is not longer, then the flow is of type III or IV. The calculations for type III and IV flow are more involved. First, the value of the parameter a_t must be determined from the following equality:

$$\sqrt{3a_t/2}[\mathrm{f}(1, a_t) - \mathrm{f}(0, a_t)] + L_s/\lambda = 0 \tag{11}$$

where

$$\mathrm{f}(\phi, a) = (3^{-1/4} - 3^{1/4})\mathrm{F}(\theta, \kappa) + 2 \cdot 3^{1/4}\mathrm{E}(\theta, \kappa) - 2 \cdot 3^{1/4}\frac{\sin\theta\sqrt{1 - \kappa^2\sin^2\theta}}{1 + \cos\theta} \tag{12}$$