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Abstract. Let p be a prime. Let A be a finite group and M be a normal subgroup of A such that all elements in $A \setminus M$ have order p. Suppose that A acts on a finite p'-group G in such a way that $C_G(M) = 1$. We show that if $C_G(x)$ is nilpotent for any $x \in A \setminus M$, then G is nilpotent. It is also proved that if A is a p-group and $C_G(x)$ is nilpotent of class at most c for any $x \in A \setminus M$, then the nilpotency class of G is bounded solely in terms of c and $|A|$.

1. Introduction

An automorphism α of a group M is called a splitting automorphism of order n if

$$\alpha^n = 1 \text{ and } x \cdot x^{\alpha} \cdot x^{\alpha^2} \cdots x^{\alpha^{n-1}} = 1$$

for all $x \in M$. It is well-known that a fixed-point-free automorphism of a finite group is a splitting automorphism. In fact the concept of splitting automorphism can be considered as a generalization of fixed-point-free action. Kegel [**7**] proved that a finite group with a splitting automorphism of prime order is nilpotent. Clearly, this is a generalization of Thompson's theorem about finite groups admitting a fixed-point-free automorphism of prime order [**14**].

Let A be a finite group and M be a normal subgroup of A. Assume that all elements in $A \setminus M$ have prime order p. Then using the identity

$$x \cdot x^{\alpha} \cdot x^{\alpha^2} \cdots x^{\alpha^{p-1}} = (x\alpha^{-1})^p \alpha^p,$$

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