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SKEW DERIVATIONS ON GENERALIZED WEYL ALGEBRAS

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ABSTRACT. A wide class of skew derivations on degree-one generalized Weyl algebras $R(a, \varphi)$ over a ring R is constructed. All these derivations are twisted by a degree-counting extensions of automorphisms of R. It is determined which of the constructed derivations are Q-skew derivations. The compatibility of these skew derivations with the natural $\mathbb{Z}$-grading of $R(a, \varphi)$ is studied. Additional classes of skew derivations are constructed for generalized Weyl algebras given by an automorphism φ of a finite order. Conditions that the central element a that forms part of the structure of $R(a, \varphi)$ need to satisfy for the orthogonality of pairs of aforementioned skew derivations are derived. In addition local nilpotency of constructed derivations is studied. General constructions are illustrated by description of all skew derivations (twisted by a degree-counting extension of the identity automorphism) of generalized Weyl algebras over the polynomial ring in one variable and with a linear polynomial as the central element.

1. Introduction

This paper is devoted to the construction of a class of skew derivations of *degree-one generalized Weyl algebras*. In ring theory generalized Weyl algebras arose in the analysis of classification of simple $sl(2)$-modules in [2] and were introduced and initially studied by Bavula in a series of papers [3], [4], [5], [6], [7], [8], and also appeared in [15]. From a different perspective, degree-one generalized Weyl algebras appeared in non-commutative algebraic geometry [20], [17] (there they were called *rank-one hyperbolic algebras*). Since their introduction these algebras have become a subject of intensive study motivated in particular by the fact that many examples of algebras arising from quantum group theory or non-commutative geometry fall into this class. Degree-one generalized Weyl algebras $R(a, \varphi)$ are obtained as polynomial extensions of a ring R by adjoining two additional generators that satisfy relations determined by an automorphism φ of R and an element a in the centre of R (see Section 2 for the precise definition), and they can be understood as generalizations of skew Laurent polynomial rings.

The motivation for this study, results of which are being presented to the reader herewith, comes from non-commutative differential geometry, where skew derivations often play the role of vector fields (cf. [18, Section 4.4]) and may be used to equip non-commutative spaces with (exterior) differential structures. Recall that a vector field on a smooth manifold X can be defined as a linear endomorphism of the algebra of smooth functions on X that satisfies the Leibniz rule. The classic formula

$$df(\chi) = \chi(f), \tag{1.1}$$

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