Journal of Algebra 489 (2017) 427–434

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Growth of monomial algebras, simple rings and free subalgebras

Be'eri Greenfeld [1]

Department of Mathematics, Bar Ilan University, Ramat Gan 5290002, Israel

ARTICLE INFO

Article history:
Received 7 October 2016
Available online 20 July 2017
Communicated by Michel Van den Bergh

Keywords:
Growth
Simple rings
Monomial algebras
Just-infinite algebras
Entropy of graded algebras
Free subalgebras
Uniformly recurrent words

ABSTRACT

We construct finitely generated simple algebras with prescribed growth types, which can be arbitrarily taken from a large variety of (super-polynomial) growth types. This (partially) answers a question raised in [9, Question 5.1].
Our construction goes through a construction of finitely generated just-infinite, primitive monomial algebras with prescribed growth type, from which we construct uniformly recurrent infinite words with subword complexity having the same growth type.
We also discuss the connection between entropy of algebras and their homomorphic images, as well as the degrees of their generators of free subalgebras.
© 2017 Published by Elsevier Inc.

1. Introduction

Fix a function $f : \mathbb{N} \to \mathbb{N}$ satisfying the following properties, which every growth function of a finitely generated associative algebra satisfies:

- Monotonely increasing, namely $f(n) < f(n+1)$ for all $n \in \mathbb{N}$;

E-mail address: beeri.greenfeld@gmail.com.
[1] The author thanks the referee for his/her useful comments on the paper.

http://dx.doi.org/10.1016/j.jalgebra.2017.07.003
0021-8693/© 2017 Published by Elsevier Inc.

- Submultiplicativity, namely $f(n+m) \leq f(n)f(m)$ for all $n, m \in \mathbb{N}$.

We think of two such functions f, g as representing the same growth type, denoted by $f \sim g$ if there exist constants $C, D > 0$ for which $f(n) \leq g(Cn) \leq f(Dn)$ for all $n \in \mathbb{N}$. This notion of equivalence is indeed reasonable since growth functions of finitely generated associative algebras are defined up to this equivalence relation.

In [16], it is shown how to realize such f as the growth type of some finitely generated monomial algebra, provided that f grows rapidly enough, namely, there is $\alpha > 0$ such that $nf(n) \leq f(\alpha n)$ for all $n \in \mathbb{N}$.

In [9], it is shown how to modify the construction of Bartholdi and Smoktunowicz to result in a primitive algebra (without further limitations on the realized function f). It is also asked in [9, Question 5.1] whether similar result could be proven for simple algebras.

In this paper we show how the basic construction from [16] can be modified to result in just-infinite algebras, namely, infinite dimensional algebras all of whose proper quotients are finite-dimensional. Since in this context it can be easily shown that just-infinitude implies primitivity, this provides another approach for the main result of [9]. These algebras are determined by infinite words, and therefore our algebras in this paper give rise to infinite words which are uniformly recurrent and have prescribed complexity growth type (which is arbitrary up to several mild constraints, see Theorem 2.1).

Uniformly recurrent words have great importance from combinatorial, dynamical and algebraic points of view [1,3,6,10,14]. It should be mentioned that in [7], Cassaigne constructs uniformly recurrent words with a large class of intermediate growth functions; the class of functions realized here is not contained in the class realized in [7].

The main application of our new construction comes in the view of a recent construction of Nekrashevych [11]. Namely, the new growth types realized here as complexity functions of uniformly recurrent words allow us to construct finitely generated simple algebras with a large variety of prescribed growth types, consisting of almost all functions realized in [16] as growth types of algebras (and later in [9] as growth types of prime and primitive algebras). This provides a partial answer to [9, Question 5.1].

Recall that for an infinite word w with letters $x_1, \dots, x_d$ we have a corresponding monomial algebra A_w generated by $x_1, \dots, x_d$, which is spanned by all monomials which occur as subwords of w. Vice versa, prime (finitely generated) monomial algebras are induced from infinite words, and many combinatorial properties of w reflect algebraically in A_w. The connection between the asymptotic subword complexity $p_w(n)$ of w and the growth $g_{A_w}(n)$ of its corresponding monomial algebra A_w is intimate: $g_{A_w}(n) = p_w(1) + \cdots + p_w(n)$ where the generating vector space is taken to be $V = \mathrm{Span}\{1, x_1, \dots, x_d\}$.

In the current construction, slight limitations must be put on the function f; however it still enables one realize arbitrary super-Gel'fand–Kirillov dimension (i.e. $f(n) \sim \exp n^r$ for arbitrary $r > 0$) or entropy, a notion which we now quickly recall.

Fix a base field k. Recall that for a graded k-algebra $R = \bigoplus_{i \in \mathbb{N}} R_i$ where $\dim_k R_i < \infty$ the *entropy*, in the sense of Newman, Schneider and Shalev [12] is defined to be:

Download English Version:

https://daneshyari.com/en/article/5771730

Download Persian Version:

https://daneshyari.com/article/5771730

Daneshyari.com