Accepted Manuscript

On vanishing class sizes in finite groups

Mariagrazia Bianchi, Julian M.A. Brough, Rachel D. Camina, Emanuele Pacifici

PII: S0021-8693(17)30400-3
DOI: http://dx.doi.org/10.1016/j.jalgebra.2017.07.007
Reference: YJABR 16305

To appear in: *Journal of Algebra*

Received date: 10 January 2017

Please cite this article in press as: M. Bianchi et al., On vanishing class sizes in finite groups, *J. Algebra* (2017), http://dx.doi.org/10.1016/j.jalgebra.2017.07.007

ON VANISHING CLASS SIZES IN FINITE GROUPS

MARIAGRAZIA BIANCHI, JULIAN M.A. BROUGH, RACHEL D. CAMINA, AND EMANUELE PACIFICI

Abstract. Let G be a finite group. An element g of G is called a *vanishing element* if there exists an irreducible character χ of G such that $\chi(g) = 0$; in this case, we say that the conjugacy class of g is a vanishing conjugacy class. In this paper, we discuss some arithmetical properties concerning the sizes of the vanishing conjugacy classes in a finite group.

1. Introduction

Many authors have investigated the relationship between the structure of a finite group G and arithmetical data connected to G. The arithmetical data can take various forms: for example, authors have considered the set of conjugacy class sizes, or the set of character degrees. The link between these different sets is also of interest, as demonstrated by the following result by C. Casolo and S. Dolfi. Suppose p and q are distinct primes and pq divides the degree of some irreducible complex character of G; then pq also divides the size of some conjugacy class of G [5, Theorem A]. As an important step in the proof of this result, the authors consider groups for which p and q both divide a conjugacy class size but pq does not, and they show that such groups are $\{p, q\}$-solvable [5, Theorem B(i)].

Recently, instead of considering all conjugacy class sizes, authors have been considering a subset of conjugacy class sizes "filtered" by the irreducible characters, namely, the set of *vanishing conjugacy class sizes* (see [3], [4], [6] and also [7] for related properties of vanishing elements). An element $g \in G$ is called a vanishing element if there exists an irreducible character χ of G such that $\chi(g) = 0$, and the conjugacy class of such an element is called a vanishing conjugacy class of G. Motivated by Casolo and Dolfi's results, we investigate some arithmetical properties of the set of vanishing conjugacy class sizes.

This context is neatly portrayed by the *prime graph* of G for class sizes. Recall that, given a finite nonempty set of positive integers X, the prime graph on X has vertex set defined as the set of all prime numbers that are divisors of some element in X, and edge set consisting of pairs $\{p, q\}$ such that pq divides some element of

2000 *Mathematics Subject Classification.* 20E45.

The first and the fourth author are partially supported by the Italian INdAM-GNSAGA, and by the PRIN 2015TW9LSR_006 "Group Theory and Applications".