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Francesco Catino, Ilaria Colazzo, Paola Stefanelli

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Semi-braces and the Yang-Baxter equation☆

Francesco CATINO[a], Ilaria COLAZZO[a], Paola STEFANELLI[a,*]

[a]*Dipartimento di Matematica e Fisica "Ennio De Giorgi"*
Università del Salento
Via Provinciale Lecce-Arnesano
73100 Lecce (Italy)

Abstract

In this paper we obtain new solutions of the Yang-Baxter equation that are left non-degenerate through left semi-braces, a generalization of braces introduced by Rump. In order to provide new solutions we introduce the asymmetric product of left semi-braces, a generalization of the semidirect product of braces, that allows us to produce several examples of left semi-braces.

Keywords: Quantum Yang-Baxter equation, set-theoretical solution, skew brace, semi-brace
2010 MSC: 16T25, 16Y99, 16N20, 81R50

1. Introduction

The Yang-Baxter equation is a basic equation of statistical mechanics that arose from a work of Yang's [18] and one of Baxter's [3]. Recall that if V is a vector space, then a function $R : V \otimes V \to V \otimes V$ is said to be a solution of the Yang-Baxter equation if

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

is satisfied, where $R_{12} = R \otimes \mathrm{id}_V$, $R_{23} = \mathrm{id}_V \otimes R$, $R_{13} = (\mathrm{id}_V \otimes \tau)(R \otimes \mathrm{id}_V)(\mathrm{id}_V \otimes \tau)$, and τ the twist map on $V \otimes V$.
In 1992 Drinfeld [8] formally proposed to study a simplified case, i.e., the set-theoretical solution of the Yang-Baxter equation. Specifically, fixed a basis X on the vector space V we may find all solutions R induced by a linear extension of a function $\mathcal{R} : X \times X \to X \times X$, where X is a basis for V. In this case, $\mathcal{R}$ is called a set-theoretic solution of the quantum Yang-Baxter equation. It is not difficult to see that if $\tau : X \times X \to X \times X$ is the twist map then a map $\mathcal{R} : X \times X \to X \times X$ if and only if the mappung $r = \tau \circ \mathcal{R}$ is a solution of the braid equation

$$r_1r_2r_1 = r_2r_1r_2$$

where $r_1 := r \times \mathrm{id}_X$ and $r_2 := \mathrm{id}_X \times r$. Later, seminal papers of Etingof, Schedler and Soloviev [9] and of Gateva-Ivanova and M. Van den Bergh in [10] laid the groundwork for the study of a particular class of these solutions, the non-degenerate involutive ones, i.e., the solutions (X, r) such that the first and the

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*Corresponding author
Email addresses: `francesco.catino@unisalento.it` (Francesco CATINO), `ilaria.colazzo@unisalento.it` (Ilaria COLAZZO), `paola.stefanelli@unisalento.it` (Paola STEFANELLI)