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# Exponential Sums and Riesz Energies

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## Abstract

We bound an exponential sum that appears in the study of irregularities of distribution (the low-frequency Fourier energy of the sum of several Dirac measures) by geometric quantities: a special case is that for all  $\{x_1, \dots, x_N\} \subset \mathbb{T}^2$ ,  $X \geq 1$  and a universal  $c > 0$

$$\sum_{i,j=1}^N \frac{X^2}{1 + X^4 \|x_i - x_j\|^4} \lesssim \sum_{\substack{k \in \mathbb{Z}^2 \\ \|k\| \leq X}} \left| \sum_{n=1}^N e^{2\pi i \langle k, x_n \rangle} \right|^2 \lesssim \sum_{i,j=1}^N X^2 e^{-cX^2 \|x_i - x_j\|^2}.$$

Since this exponential sum is intimately tied to rather subtle distribution properties of the points, we obtain nonlocal structural statements for near-minimizers of the Riesz-type energy. For  $X \gtrsim N^{1/2}$  both upper and lower bound match for maximally-separated point sets satisfying  $\|x_i - x_j\| \gtrsim N^{-1/2}$ .

*Keywords:* Riesz energy, Discrepancy, Exponential sums, Fejér kernel  
*2010 MSC:* 11L07, 42B05, 52C35 (primary), 74G65 (secondary)

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## 1. Introduction and Main Results

### 1.1. Introduction

Let  $\{x_1, \dots, x_N\} \subset \mathbb{T}^2$  (throughout this paper normalized to  $\mathbb{T}^2 \cong [0, 1]^2$ ). Montgomery's theorem [27] (see also Beck [4, 5]) is a classical example of an irregularity of distribution phenomenon: there exists a disk  $D \subset \mathbb{T}^2$  with radius  $1/4$  or  $1/2$  such that the number of elements in the disk substantially deviates from its expectation

$$|\#\{1 \leq i \leq N : x_i \in D\} - N|D|| \gtrsim N^{1/4}.$$

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