Accepted Manuscript

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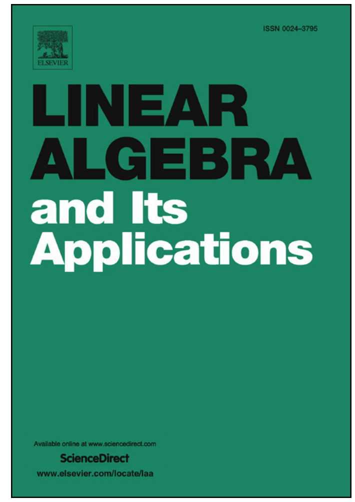

PII: S0024-3795(17)30565-7
DOI: https://doi.org/10.1016/j.laa.2017.09.031
Reference: LAA 14337

To appear in: *Linear Algebra and its Applications*

Received date: 22 May 2017
Accepted date: 27 September 2017

Please cite this article in press as: I.-P. Kim, A.R. Kräuter, Decompositions of a matrix by means of its dual matrices with applications, *Linear Algebra Appl.* (2017), https://doi.org/10.1016/j.laa.2017.09.031

Decompositions of a matrix by means of its dual matrices with applications☆

Ik-Pyo Kim[a,*], Arnold R. Kräuter[b]

[a] *Department of Mathematics Education, Daegu University, Gyeongbuk, 38453, Republic of Korea*

[b] *Department für Mathematik und Informationstechnologie, Lehrstuhl für Mathematik und Statistik, Montanuniversität Leoben, Franz-Josef-Strasse 18, 8700 Leoben, Austria*

Abstract

We introduce the notion of dual matrices of an infinite matrix A, which are defined by the dual sequences of rows of A and naturally connected to the Pascal matrix $P = [\binom{i}{j}]$ $(i, j = 0, 1, 2, \ldots)$. We present the Cholesky decomposition of the symmetric Pascal matrix by means of its dual matrix. Decompositions of a Vandermonde matrix are used to obtain variants of the Lagrange interpolation polynomial of degree $\leq n$ that passes through the $n + 1$ points (i, q_i) for $i = 0, 1, \ldots, n$.

Keywords: Dual sequence, Dual matrix, Vandermonde matrix, Hankel matrix, Toeplitz matrix, Lagrange interpolation polynomial
2010 MSC: 15B05, 11B39, 11B65

1. Introduction

Let $\mathbf{R}^\infty$ denote the infinite dimensional real vector space consisting of all real sequences $\mathbf{a} = (a_0, a_1, a_2, \ldots)^T$, and let $\Delta\mathbf{a}$ denote the *difference sequence* of $\mathbf{a}$, defined by:

$$\Delta\mathbf{a} = (\Delta a_0, \Delta a_1, \Delta a_2, \ldots)^T,$$

where $\Delta a_i = a_{i+1} - a_i$ for each $i = 0, 1, 2, \ldots$ Let $\Delta^k\mathbf{a} = (\Delta^k a_0, \Delta^k a_1, \Delta^k a_2, \ldots)^T$, $k = 0, 1, 2, \ldots$, be the kth difference sequence defined inductively by $\Delta^k\mathbf{a} = \Delta(\Delta^{k-1}\mathbf{a})$, where $\Delta^0\mathbf{a} = \mathbf{a}$. For a sequence $\mathbf{a} = (a_0, a_1, a_2, \ldots)^T \in \mathbf{R}^\infty$, let $\mathbb{M}^{\mathbf{a}}$ denote the *difference matrix* of $\mathbf{a}$ defined by:

$$\mathbb{M}^{\mathbf{a}} = \begin{bmatrix} \mathbf{a}^T \\ (\Delta\mathbf{a})^T \\ (\Delta^2\mathbf{a})^T \\ \vdots \end{bmatrix} \tag{1.1}$$

☆Research supported by Daegu University Research Grant 2014
*Corresponding author
Email addresses: kimikpyo@daegu.ac.kr (Ik-Pyo Kim), arnold.kraeuter@unileoben.ac.at (Arnold R. Kräuter)