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Generic skew-symmetric matrix polynomials with fixed rank and fixed odd grade ☆

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ABSTRACT

We show that the set of $m \times m$ complex skew-symmetric matrix polynomials of odd grade d, i.e., of degree at most d, and (normal) rank at most $2r$ is the closure of the single set of matrix polynomials with the certain, explicitly described, complete eigenstructure. This complete eigenstructure corresponds to the most generic $m \times m$ complex skew-symmetric matrix polynomials of odd grade d and rank at most $2r$. In particular, this result includes the case of skew-symmetric matrix pencils ($d = 1$).

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1. Introduction

The structure of the sets of matrix pencils and of matrix polynomials with fixed grade and fixed rank is not trivial and, as a consequence, only recently has been investigated in the literature [10,13,22], with the main purpose of providing reasonably simple descriptions of these sets as the closures of certain "generic sets" of pencils and matrix polynomials that can be easily described in terms of their eigenstructures or in terms of certain parameterizations. One motivation for this type of research comes from its applications in the study of the effect of low rank perturbations on the spectral information of pencils and matrix polynomials, since these problems have received considerable attention in the last years. See for instance the references [2–4,9,12,18,45] for different low rank perturbation problems related to matrix pencils, and the particular study in [11] on certain low rank perturbations of matrix polynomials. Moreover, low rank perturbations of matrix pencils have been applied recently to some classical problems as the eigenvalue placement problem [32] or the estimation of the distance of a regular pencil to the nearest singular pencil [44]. Finally, from a more theoretical perspective, the study of the sets of matrix pencils and of matrix polynomials with fixed grade and fixed rank generalizes classical studies [50] on the algebraic structure of the set of $n \times n$ singular pencils, i.e., those whose rank is at most $n - 1$.

As can be observed in the titles of several of the references mentioned in the paragraph above, some of the problems concerning low rank perturbations of matrix pencils involve pencils with particular structures (symmetric, Hermitian, palindromic, skew-symmetric, alternating, etc), since the pencils and matrix polynomials arising in applications have often such particular structures [39]. In this scenario, it is natural to consider the extension of the results in [10,13,22] to the structured setting, with the aim of providing simple descriptions of sets of structured matrix pencils and of structured matrix polynomials with fixed grade and fixed rank in the case of the structures appearing in applications. The results in this paper can be seen as the first step available in the literature towards the solution of this ambitious and wide problem. In particular, we prove that the set of $m \times m$ complex skew-symmetric matrix polynomials of odd grade d, i.e., of degree at most d, and (normal) rank at most $2r$ is the closure of the single set of matrix polynomials with the certain, explicitly described, complete eigenstructure, which is termed as the *unique generic eigenstructure* of this set of skew-symmetric matrix polynomials of odd grade d and rank at most $2r$. By taking $d = 1$, we obtain that such uniqueness also holds for skew-symmetric pencils. This result greatly simplifies the description of the set of skew-symmetric polynomials of fixed odd grade and fixed rank and illustrates how strong can be the effect of imposing a structure on this type of problems, since the corresponding unstructured problem has many more generic complete eigenstructures. More precisely, it has been proved in [10,22] that there are $rd+1$ generic complete eigenstructures in the set of $m \times m$ complex *general (unstructured)* matrix polynomials of odd grade d and rank at most r.