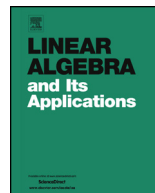




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# Linear Algebra and its Applications

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## The first two largest spectral radii of uniform supertrees with given diameter <sup>☆</sup>



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### ABSTRACT

A supertree is a connected and acyclic hypergraph. Let  $\mathbb{S}(m, d, k)$  be the set of  $k$ -uniform supertrees with  $m$  edges and diameter  $d$ . In this paper, we determine the supertrees with the first two largest spectral radii among all supertrees in  $\mathbb{S}(m, d, k)$  for  $3 \leq d \leq m - 1$ .

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### 1. Introduction

A hypergraph  $H = (V, E)$  consists of a vertex set  $V = V(H)$  and an edge set  $E = E(H)$ , where  $V = V(H)$  is nonempty and each edge  $e \in E(H)$  is a subset of  $V(H)$  containing at least two elements. We say that a hypergraph  $H$  is  $k$ -uniform if every edge has size  $k$ . A simple graph is a 2-uniform hypergraph.

For  $u, v \in V(H)$ , a walk from  $u$  to  $v$  in  $H$  is defined to be a sequence of vertices and edges  $v_1, e_1, v_2, \dots, e_d, v_{d+1}$  with  $v_1 = u$  and  $v_{d+1} = v$  such that edge  $e_i$  contains vertices  $v_i$  and  $v_{i+1}$ , and  $v_i \neq v_{i+1}$  for  $i = 1, 2, \dots, d$ . The value  $d$  is the length of this walk. A path is a walk with all  $v_i$  distinct and all  $e_i$  distinct. A cycle is a walk containing at least two edges, all  $e_i$  are distinct and all  $v_i$  are distinct except  $v_1 = v_{d+1}$ . A hypergraph is connected if for any pair of vertices, there is a path which connects these vertices; it is not connected otherwise. The distance  $d(u, v) = d_H(u, v)$  between two vertices  $u$  and  $v$  is the minimum length of a path which connects  $u$  and  $v$ . The diameter  $d(H)$  of  $H$  is defined by  $d(H) = \max\{d(u, v) : u, v \in V(H)\}$ .

For a vertex  $v \in V(H)$ , the degree  $d(v) = d_H(v)$  of  $v$  is the number of edges containing  $v$  of  $H$ . A vertex of degree one of  $H$  is called a pendant vertex of  $H$ . In a  $k$ -uniform hypergraph, an edge  $e$  is called a pendant edge if  $e$  contains exactly  $k - 1$  vertices of degree one. If  $e$  is not a pendant edge, then it is called a non-pendant edge. An edge  $e$  is said to be incident to a vertex  $v$  if  $v \in e$ .

The concept of tensor eigenvalues was independently proposed in [9,13]. More details on eigenvalues and tensors can be found in [3,14,18].

**Definition 1.1.** An order  $k$  dimension  $n$  tensor  $\mathcal{A} = (a_{i_1 i_2 \dots i_k}) \in \mathbb{C}^{n \times n \times \dots \times n}$  is a multidimensional array with  $n^k$  entries, where  $i_j \in [n] = \{1, 2, \dots, n\}$  for each  $j = 1, 2, \dots, k$ .

**Definition 1.2.** Let  $H = (V, E)$  be a  $k$ -uniform hypergraph on  $n$  vertices. The adjacency tensor  $\mathcal{A} = \mathcal{A}(H)$  of  $H$  is defined as the order  $k$  dimension  $n$  tensor with entries  $a_{i_1 i_2 \dots i_k}$  such that

$$a_{i_1 i_2 \dots i_k} = \begin{cases} \frac{1}{(k-1)!}, & \{i_1, i_2, \dots, i_k\} \in E(H), \\ 0, & \text{otherwise.} \end{cases}$$

**Definition 1.3.** ([13]) Let  $\mathcal{A}$  be an order  $k$  dimension  $n$  tensor, and  $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{C}^n$  be a column vector of dimension  $n$ . Then  $\mathcal{A}x^{k-1}$  is defined to be a vector in  $\mathbb{C}^n$  whose  $i$ th component is:

$$(\mathcal{A}x^{k-1})_i = \sum_{i_2, \dots, i_k=1}^n a_{i i_2 \dots i_k} x_{i_2} \dots x_{i_k}, \quad (i = 1, 2, \dots, n).$$

Let  $x^{[r]} = (x_1^r, x_2^r, \dots, x_n^r)^T \in \mathbb{C}^n$ . Then a number  $\lambda \in \mathbb{C}$  is called an eigenvalue of the tensor  $\mathcal{A}$  if there exists a nonzero vector  $x \in \mathbb{C}^n$  such that

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