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The first two largest spectral radii of uniform supertrees with given diameter ☆

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ABSTRACT

A supertree is a connected and acyclic hypergraph. Let $\mathbb{S}(m,d,k)$ be the set of k-uniform supertrees with m edges and diameter d. In this paper, we determine the supertrees with the first two largest spectral radii among all supertrees in $\mathbb{S}(m,d,k)$ for $3 \le d \le m-1$.

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1. Introduction

A hypergraph $H = (V, E)$ consists of a vertex set $V = V(H)$ and an edge set $E = E(H)$, where $V = V(H)$ is nonempty and each edge $e \in E(H)$ is a subset of $V(H)$ containing at least two elements. We say that a hypergraph H is k-uniform if every edge has size k. A simple graph is a 2-uniform hypergraph.

For $u, v \in V(H)$, a walk from u to v in H is defined to be a sequence of vertices and edges $v_1, e_1, v_2, \dots, e_d, v_{d+1}$ with $v_1 = u$ and $v_{d+1} = v$ such that edge e_i contains vertices v_i and v_{i+1}, and $v_i \neq v_{i+1}$ for $i = 1, 2, \dots, d$. The value d is the length of this walk. A path is a walk with all v_i distinct and all e_i distinct. A cycle is a walk containing at least two edges, all e_i are distinct and all v_i are distinct except $v_1 = v_{d+1}$. A hypergraph is connected if for any pair of vertices, there is a path which connects these vertices; it is not connected otherwise. The distance $d(u, v) = d_H(u, v)$ between two vertices u and v is the minimum length of a path which connects u and v. The diameter $d(H)$ of H is defined by $d(H) = \max\{d(u, v) : u, v \in V(H)\}$.

For a vertex $v \in V(H)$, the degree $d(v) = d_H(v)$ of v is the number of edges containing v of H. A vertex of degree one of H is called a pendant vertex of H. In a k-uniform hypergraph, an edge e is called a pendant edge if e contains exactly $k - 1$ vertices of degree one. If e is not a pendant edge, then it is called a non-pendant edge. An edge e is said to be incident to a vertex v if $v \in e$.

The concept of tensor eigenvalues was independently proposed in [9,13]. More details on eigenvalues and tensors can be found in [3,14,18].

Definition 1.1. An order k dimension n tensor $\mathcal{A} = (a_{i_1 i_2 \dots i_k}) \in \mathbb{C}^{n \times n \times \dots \times n}$ is a multidimensional array with n^k entries, where $i_j \in [n] = \{1, 2, \dots, n\}$ for each $j = 1, 2, \cdots, k$.

Definition 1.2. Let $H = (V, E)$ be a k-uniform hypergraph on n vertices. The adjacency tensor $\mathcal{A} = \mathcal{A}(H)$ of H is defined as the order k dimension n tensor with entries $a_{i_1 i_2 \dots i_k}$ such that

$$a_{i_1 i_2 \dots i_k} = \begin{cases} \frac{1}{(k-1)!}, & \{i_1, i_2, \dots, i_k\} \in E(H), \\ 0, & \text{otherwise.} \end{cases}$$

Definition 1.3. ([13]) Let $\mathcal{A}$ be an order k dimension n tensor, and $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{C}^n$ be a column vector of dimension n. Then $\mathcal{A}x^{k-1}$ is defined to be a vector in $\mathbb{C}^n$ whose ith component is:

$$(\mathcal{A}x^{k-1})_i = \sum_{i_2, \dots, i_k = 1}^{n} a_{i i_2 \dots i_k} x_{i_2} \dots x_{i_k}, \ (i = 1, 2, \dots, n).$$

Let $x^{[r]} = (x_1^r, x_2^r, \dots, x_n^r)^T \in \mathbb{C}^n$. Then a number $\lambda \in \mathbb{C}$ is called an eigenvalue of the tensor $\mathcal{A}$ if there exists a nonzero vector $x \in \mathbb{C}^n$ such that