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Primal-dual potential reduction algorithm for symmetric programming problems with nonlinear objective functions

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ABSTRACT

We consider a primal-dual potential reduction algorithm for nonlinear convex optimization problems over symmetric cones. The same complexity estimates as in the case of the linear objective function are obtained provided a certain nonlinear system of equations can be solved with a given accuracy. This generalizes the result of K. Kortanek, F. Potra and Y. Ye [7]. We further introduce a generalized Nesterov–Todd direction and show how it can be used to achieve a required accuracy (by solving the linearization of above mentioned nonlinear system) for a class of nonlinear convex functions satisfying scaling Lipschitz condition. This result is a far-reaching generalization of results of F. Potra, Y. Ye and J. Zhu [8], [9]. Finally, we show that a class of functions (which contains quantum entropy function) satisfies scaling Lipschitz condition.

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1. Introduction

A substantial amount of optimization problems arising in quantum information theory, quantum statistical physics, information geometry, machine learning and other areas (for the impressive list of various applications see [1]) requires dealing with the so-called quantum or von Neumann entropy which is the function of the form Tr(XLn(X)), where X is positive semi-definite complex Hermitian matrix. In [5] we introduced a class of functions (which contains the quantum entropy) which is compatible (in the sense of [10]) with the standard logarithmic barrier on the cone of invertible squares of a Euclidean Jordan algebra. This, in principle, allows one to use certain types of interior-point algorithms for solving optimization problems over symmetric cones involving quantum entropy.

In present paper we develop a primal-dual potential reduction algorithm for a class of nonlinear convex objective functions (which contains the quantum entropy) which provides an efficient way of dealing with nonlinear convex optimization problems over symmetric cones. In particular, the complexity estimates for the algorithm are (qualitatively) the same as in the case of linear objective functions [3]. Our results are far-reaching generalizations of those of K. Kortanek, F. Potra, Y. Ye and J. Zhu.

The plan of the paper is as follows. In section 2 we briefly recall relevant notations and results from the theory of Euclidean Jordan algebras (following [2]). In section 3 we formulate a version of a nonlinear complementarity problem based on the concept of a monotone manifold and describe a conceptual primal-dual algorithm for (approximately) solving it. In section 4 we develop a potential-reduction algorithm for a general nonlinear convex objective function for optimization problems over symmetric cones. In section 5 a (generalized) primal-dual Nesterov–Todd direction is introduced. It is shown how to specify the potential reduction algorithm for a class of nonlinear convex objective functions satisfying the so-called scaled Lipschitz condition [9]. This section heavily relies on [3]. Finally, in section 6 we show that the class of functions introduced in [5] satisfies the scaling Lipschitz condition.

2. Jordan-algebraic concepts

We adhere to the notation of an excellent book [2].

Let $\mathbf{F}$ be the field $\mathbf{R}$ or $\mathbf{C}$. A vector space V over $\mathbf{F}$ is called an algebra over $\mathbf{F}$ if a bilinear mapping $(x, y) \to xy$ from $V \times V$ into V is defined. For an element x in V let $L(x) : V \to V$ be the linear map such that

$$L(x)y = xy.$$

An algebra V over $\mathbf{F}$ is a Jordan algebra if

$$xy = yx, x(x^2y) = x^2(xy), \forall x, y \in V.$$